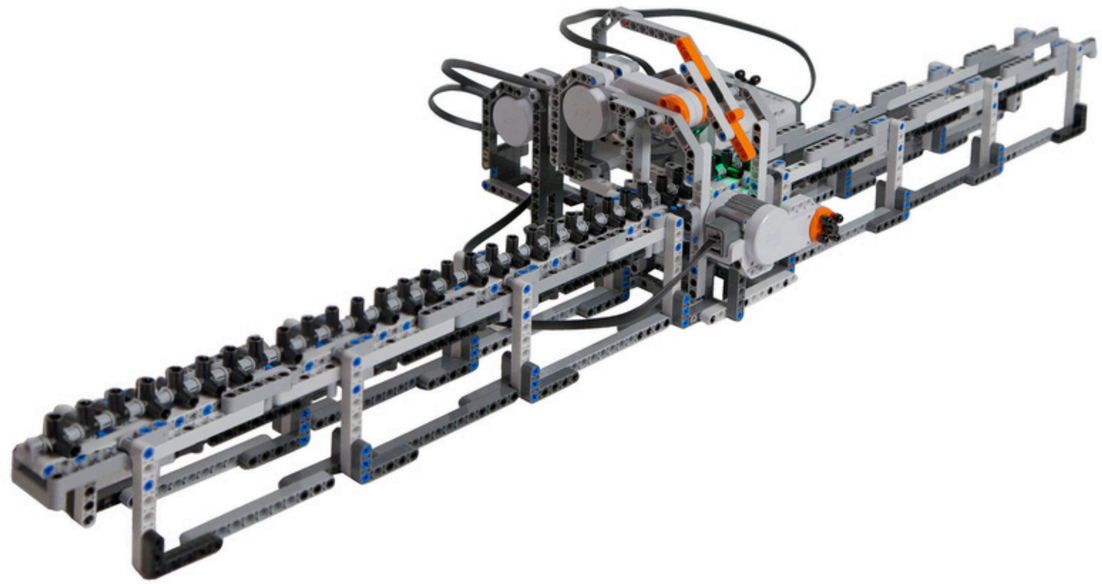
Compositional Sequentialization of Periodic Programs*

Soonho Kong
soonhok@cs.cmu.edu
Carnegie Mellon University

*Work with Sagar Chaki(SEI/CMU), Arie Gurfinkel(SEI/CMU), and Ofer Strichman(Technion - Israel Institute of Technology)

Target of Verification:

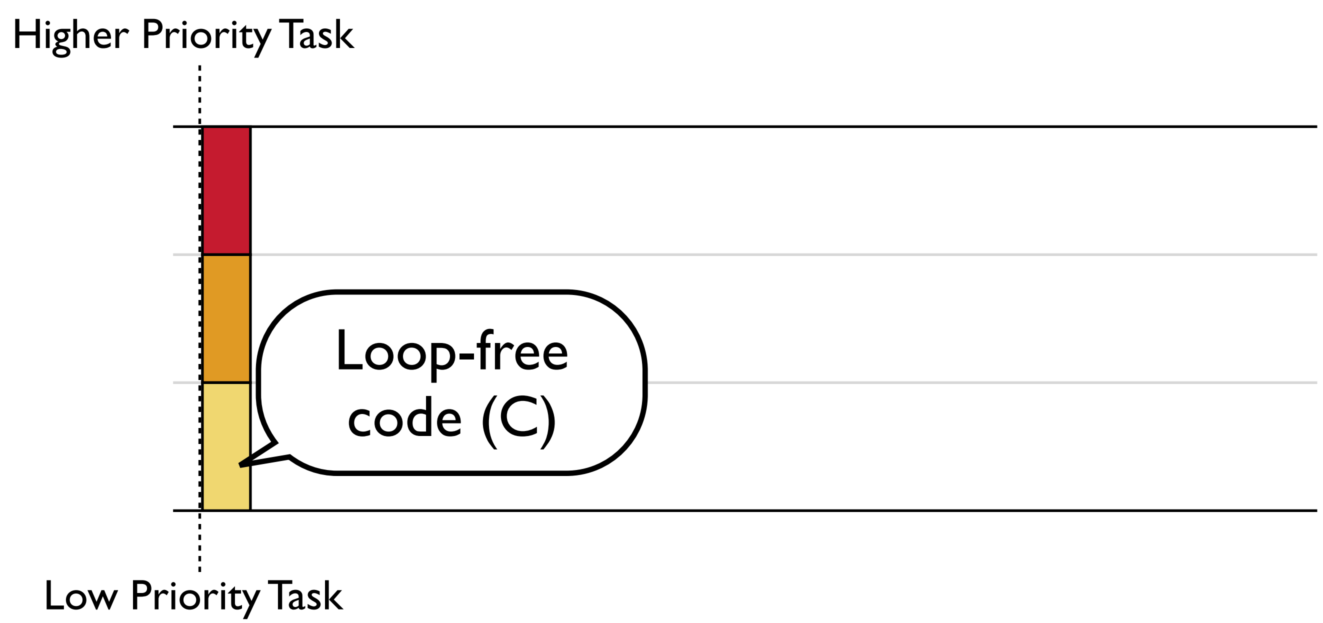
Periodic Programs



Task $\tau = (I, T, P, C, A)$

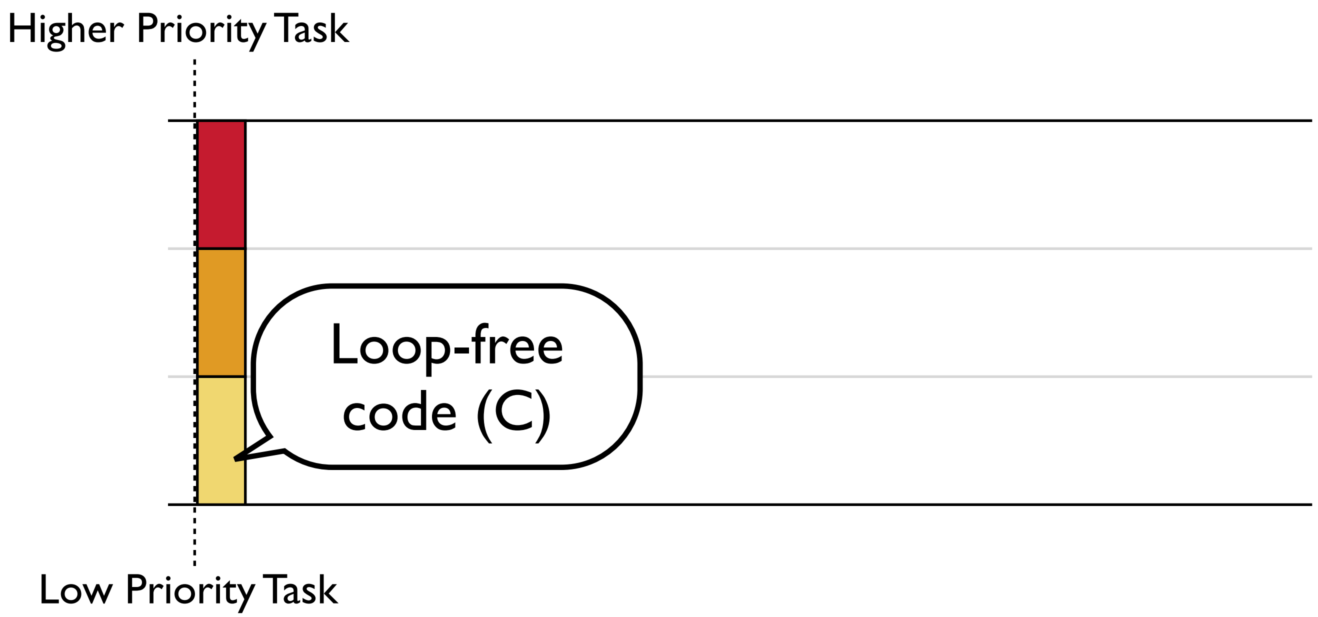
Priority

Task $\tau = (I, T, P, C, A)$



TaskBody

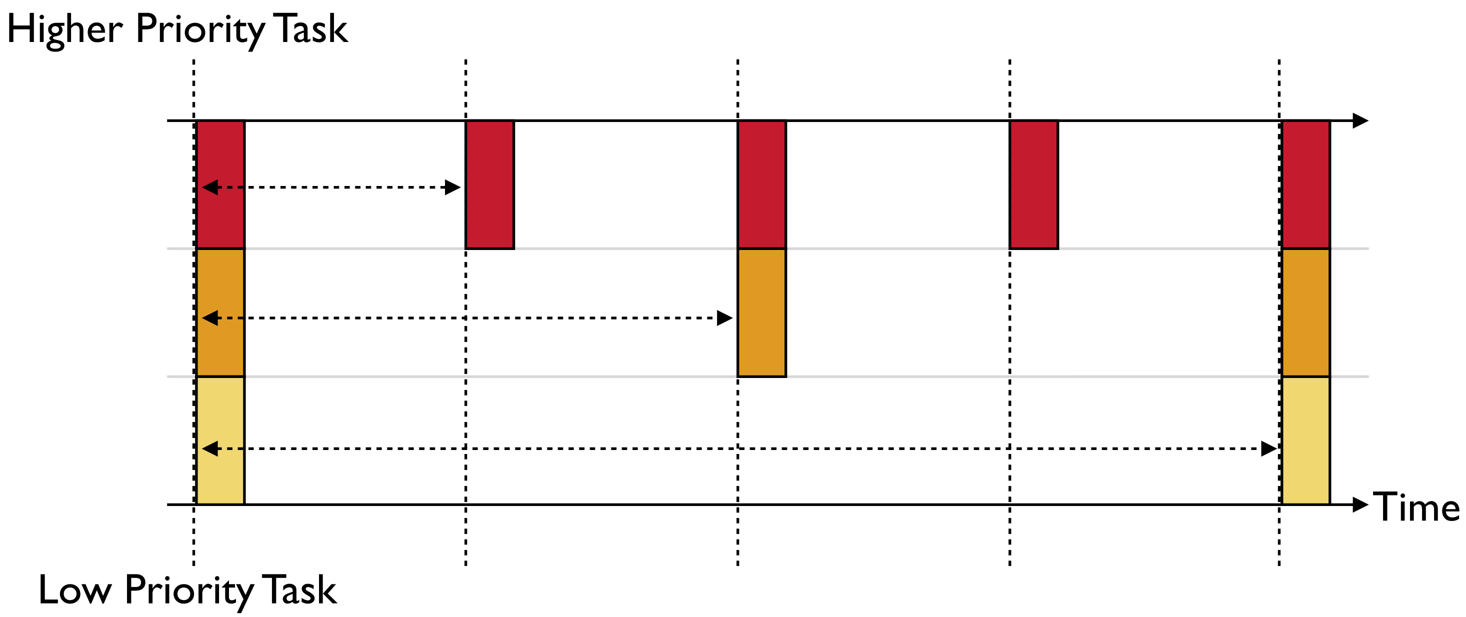
Task $\tau = (I, T, P, C, A)$

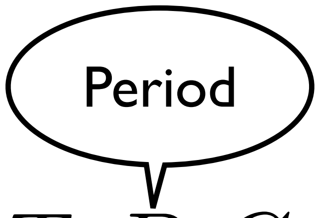


TaskBody

$$\text{Task } \tau = (I, T, P, C, A)$$

```
TASK(Controller)
{
  int old_state = state;
  if(R(need_to_run_nxtbg)) {
    bg_nxtcolorsensor(true);
    W(need_to_run_nxtbg, false);
  }

  switch (TM_mode) {
  case TM_CALIBRATE:
    W(threshold, calibrate());
    if(R(threshold) > 0) {
      TM_mode = TM_INIT;
    }
    break;

  case TM_INIT:
    /* initialize internal variable */
    init();
    TM_mode = TM_HALT;
    break;

  case TM_OPERATION:
    switch(C_state) {
    case C_READ:
      if(R(need_to_read)) {
        if(nxt_motor_get_count(READ_MOTOR) < READ_REV && R(R_state) == RE
          W(R_state, READ_MOVE_HEADER_FORWARD);
        } else if(nxt_motor_get_count(READ_MOTOR) >= READ_REV && R(R_state
          W(R_state, READ_SENSOR);
        }
      } else {
        W(R_state, READ_IDLE);
        C_state = C_TRANS;
      }
      break;

    case C_TRANS:
      old_state = state;
      if(transition(state, R(input))) {
        TM_mode = TM_HALT;
      } else {
        C_state = C_WRITE;
      }
      break;

    case C_WRITE:

      /* Check if we need to chagne the bit */
      if(R(input) != R(output)) {
        /* Check the header and move it back if necessary */
```





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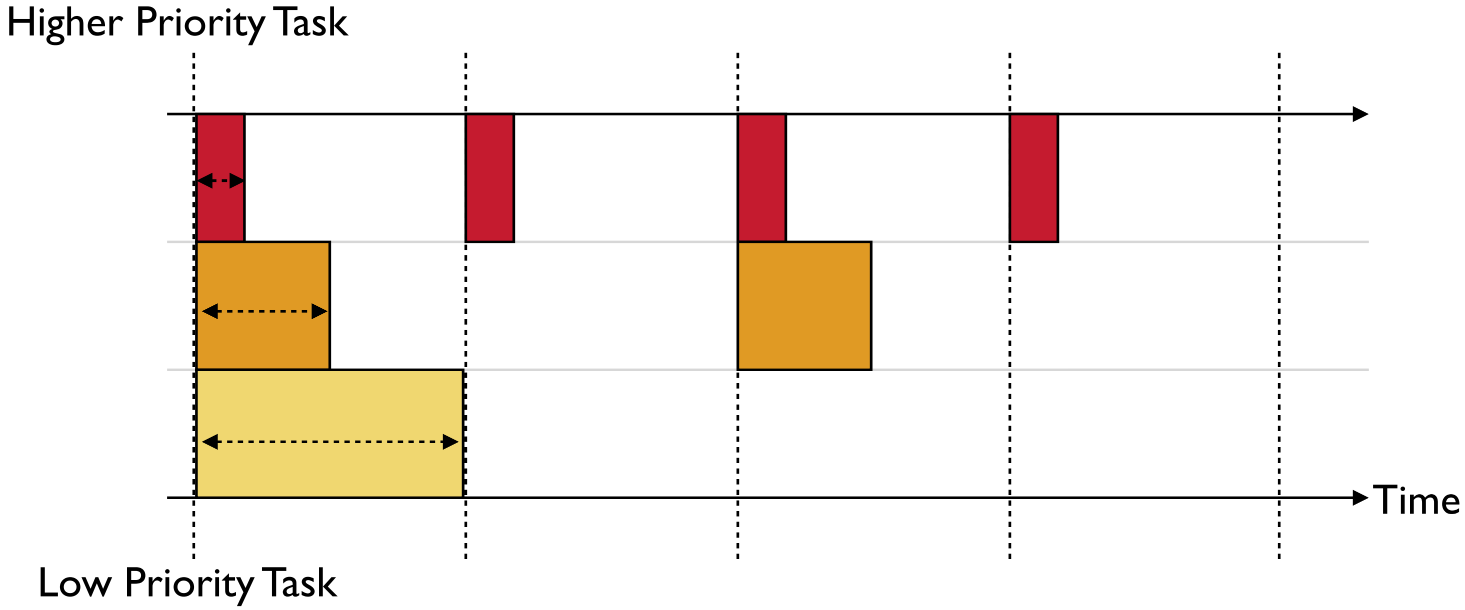

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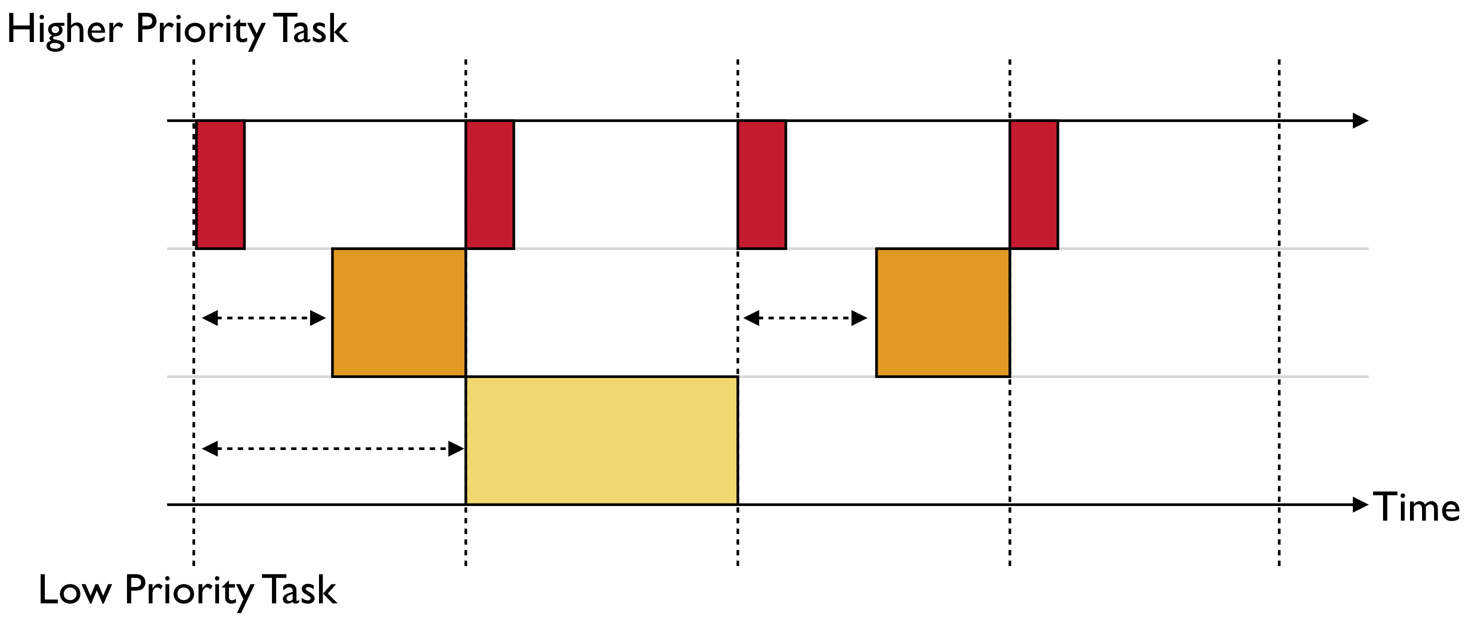

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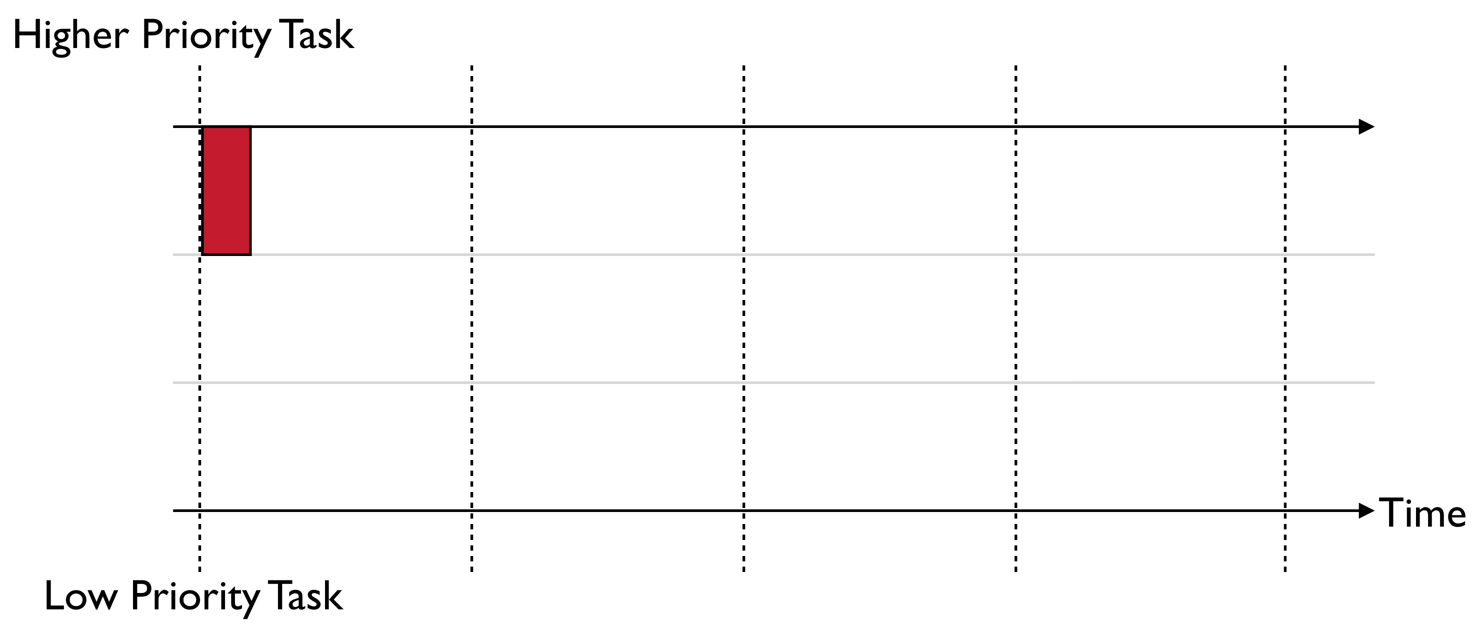
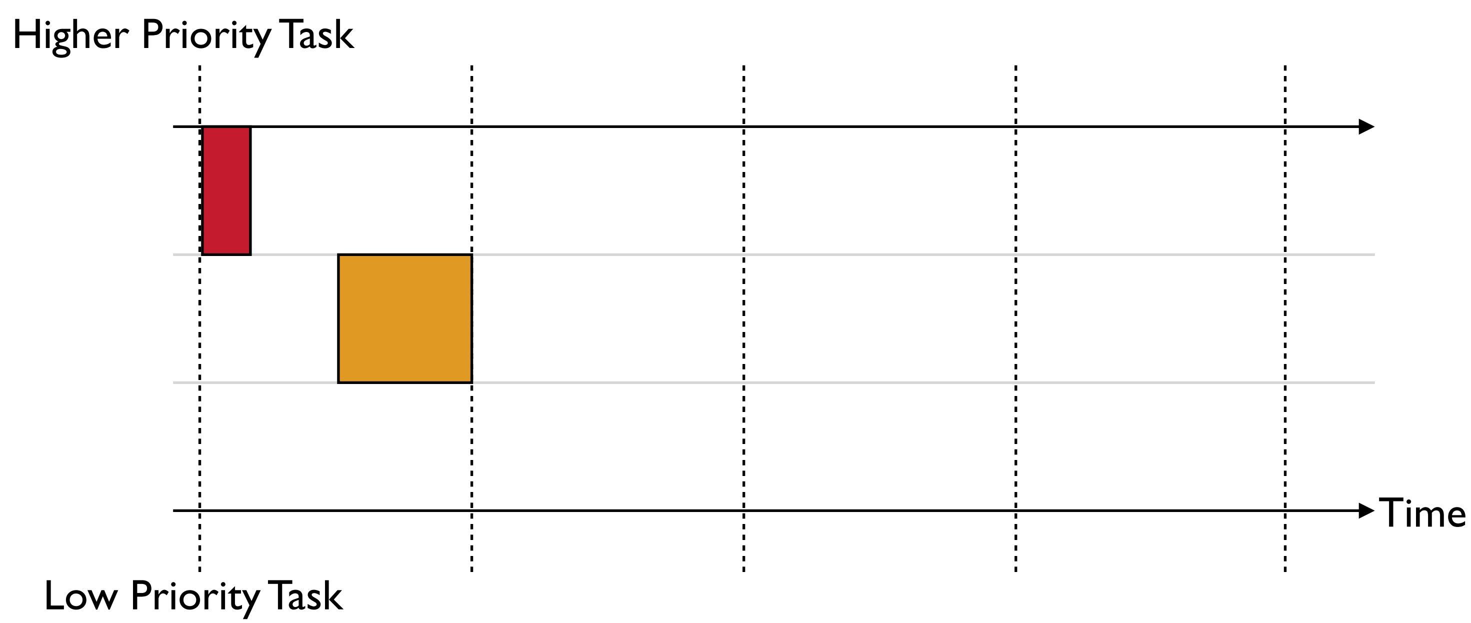


Preemptive Fixed Priority-based Scheduling



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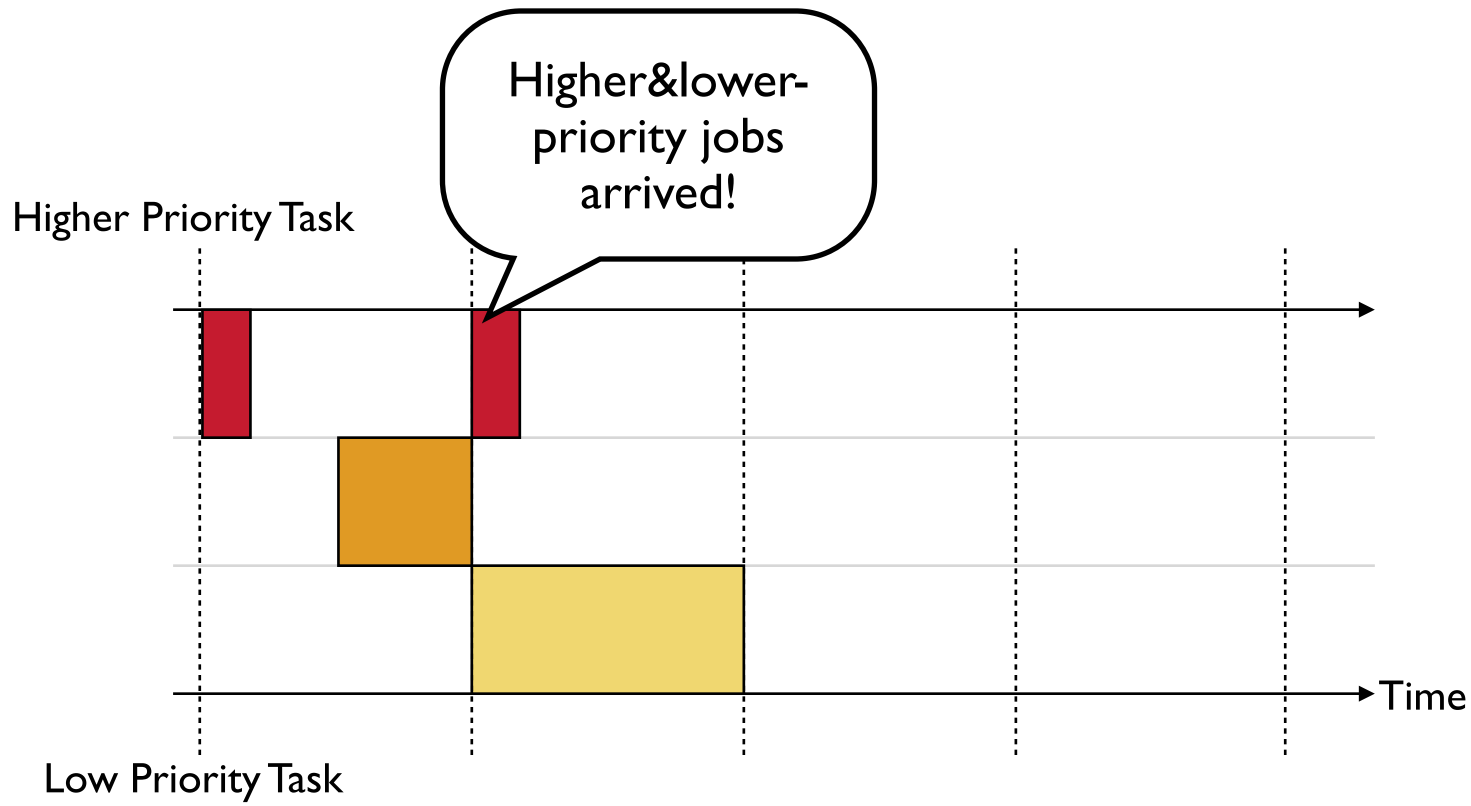

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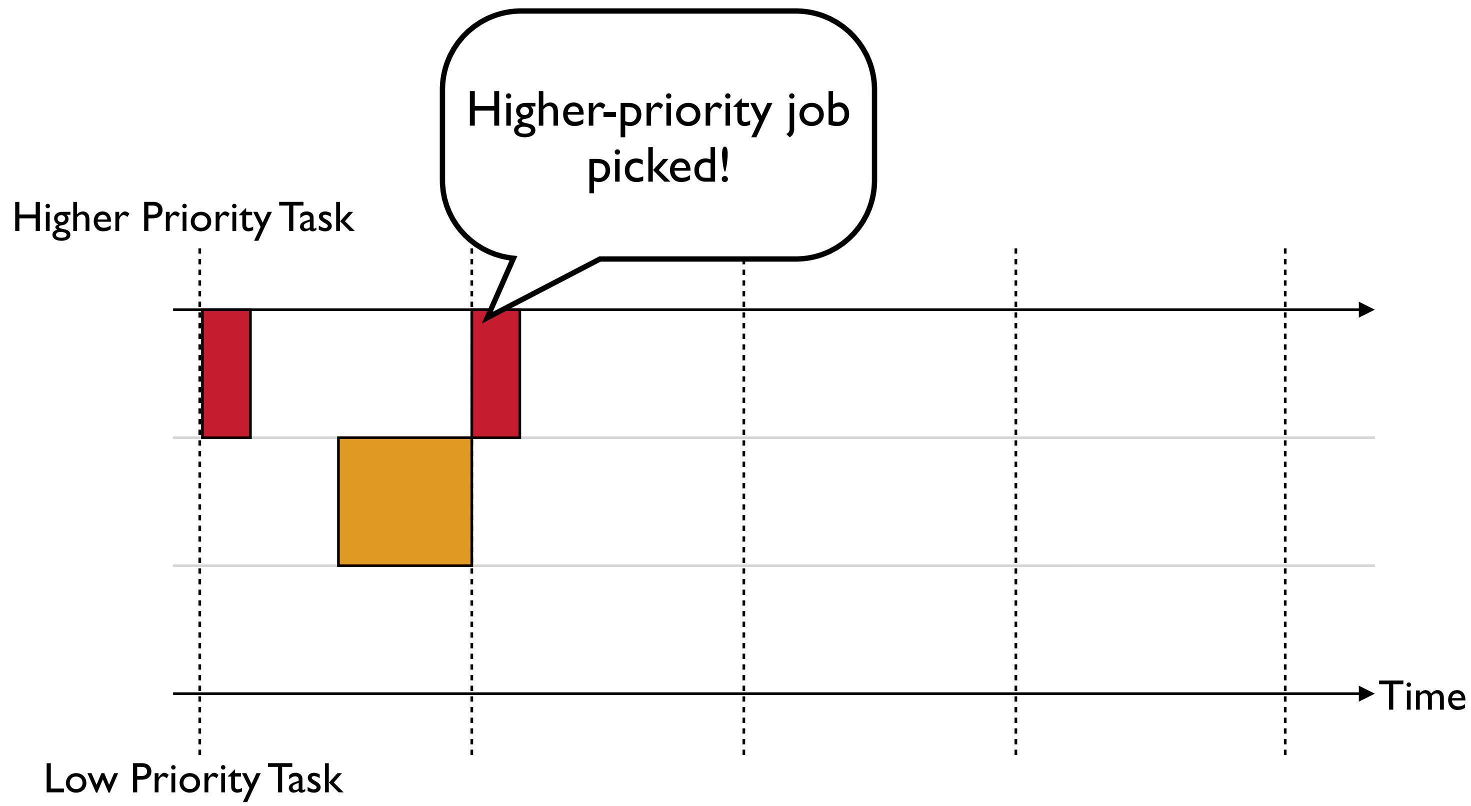

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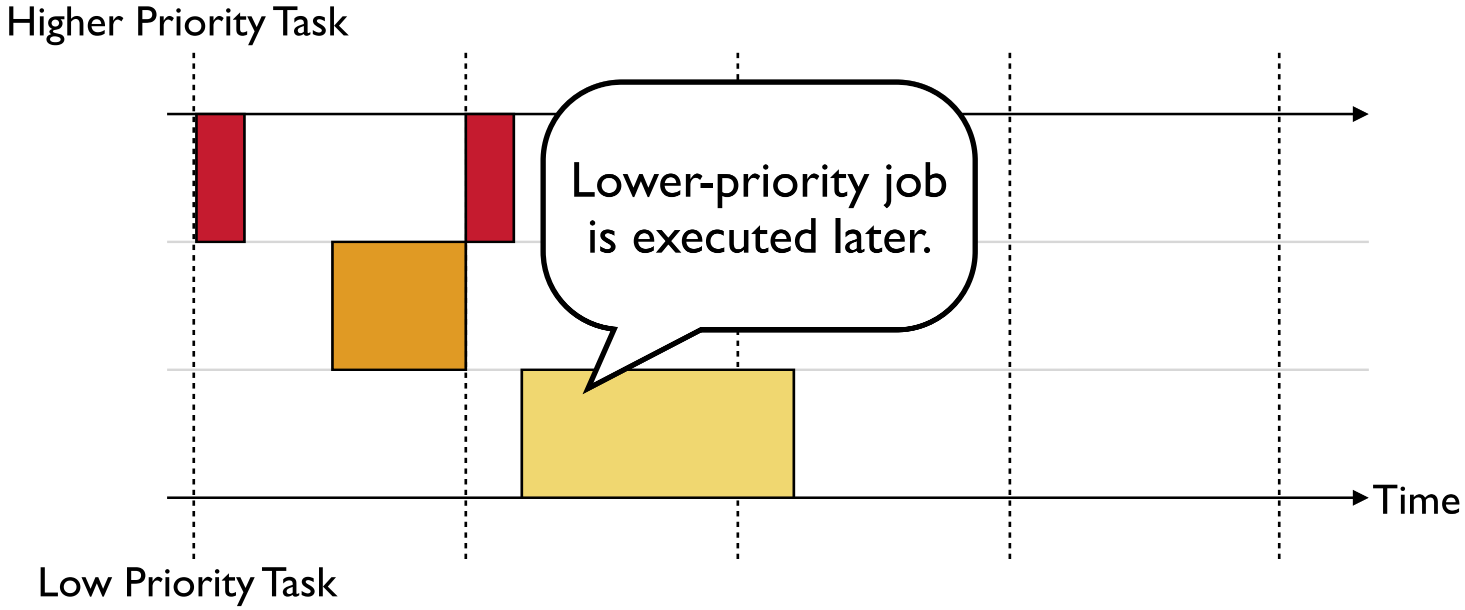

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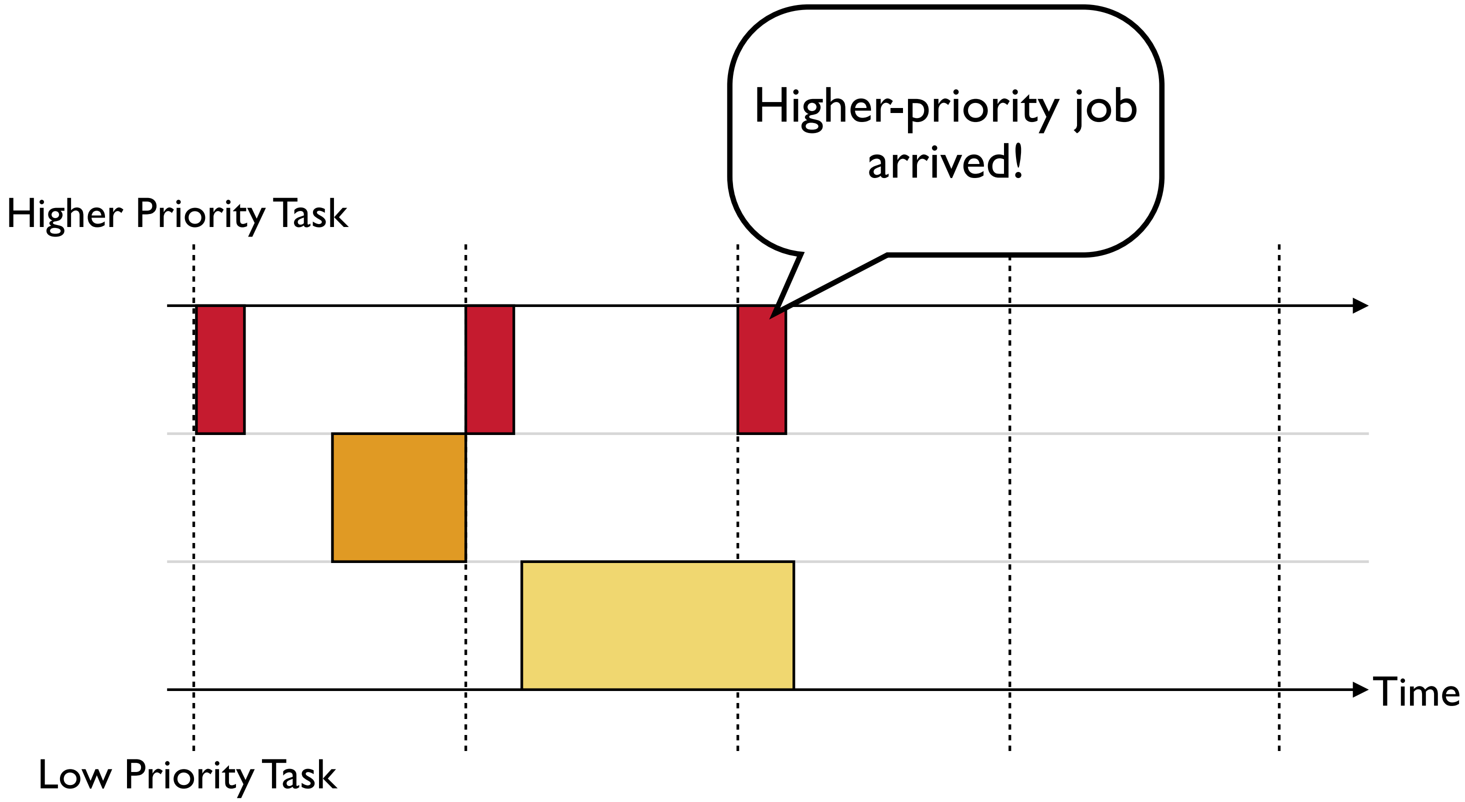

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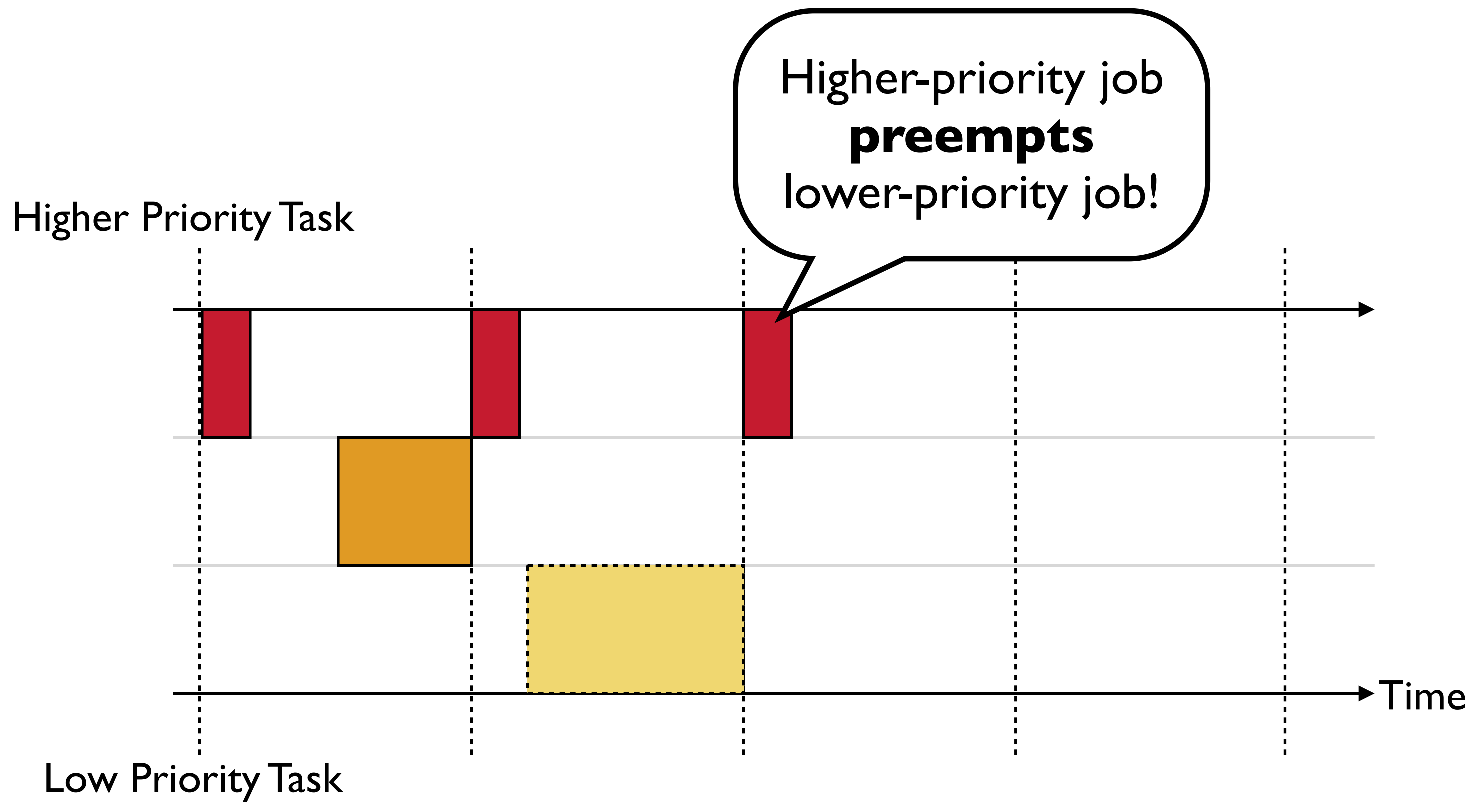

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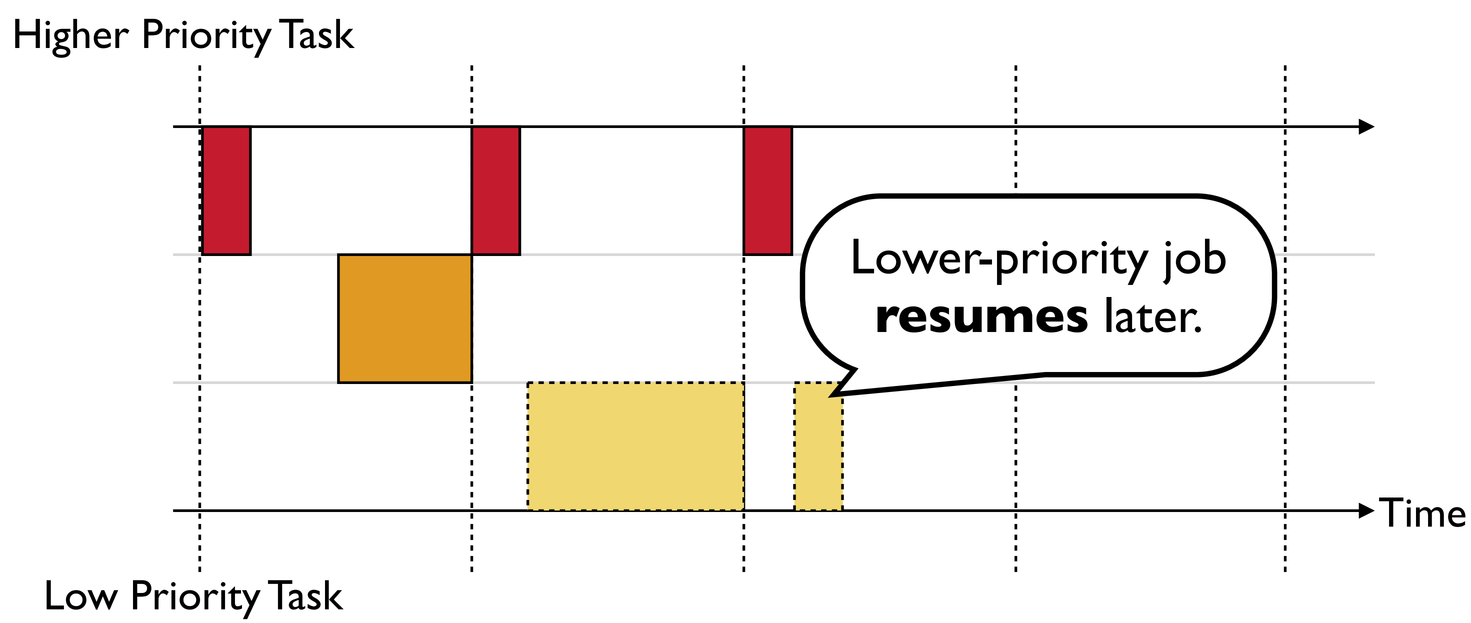

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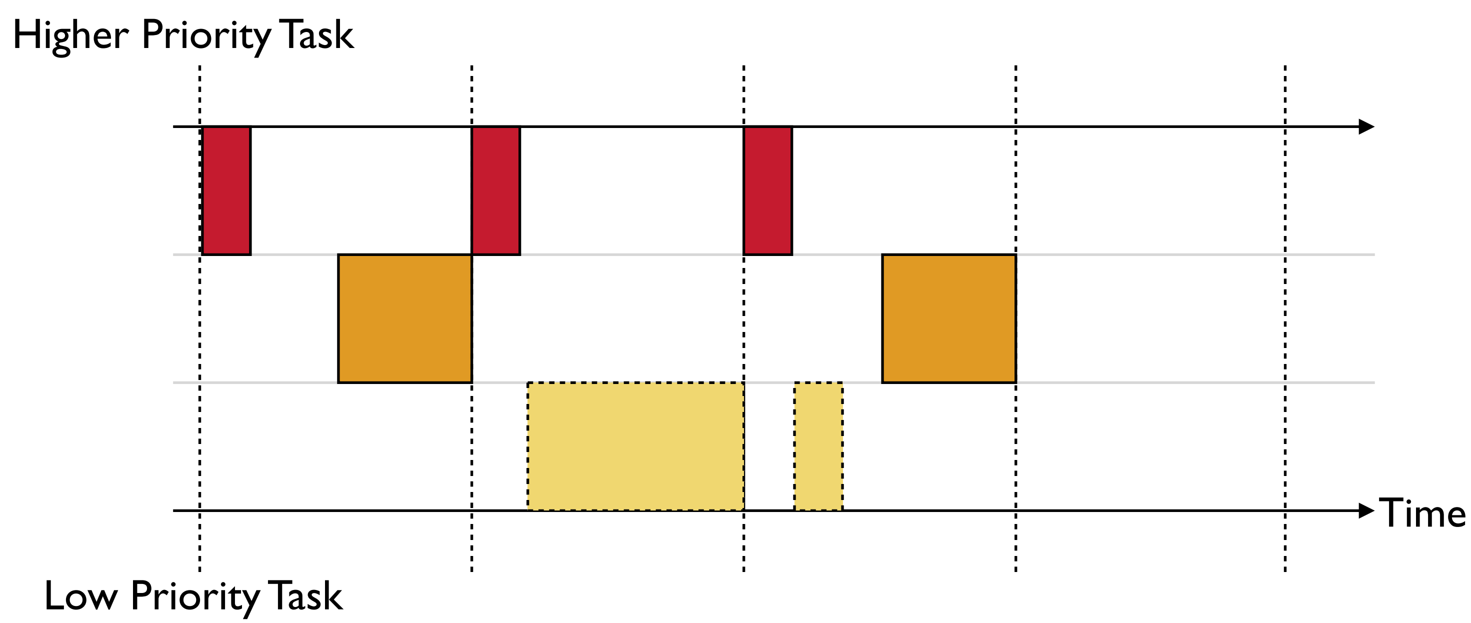

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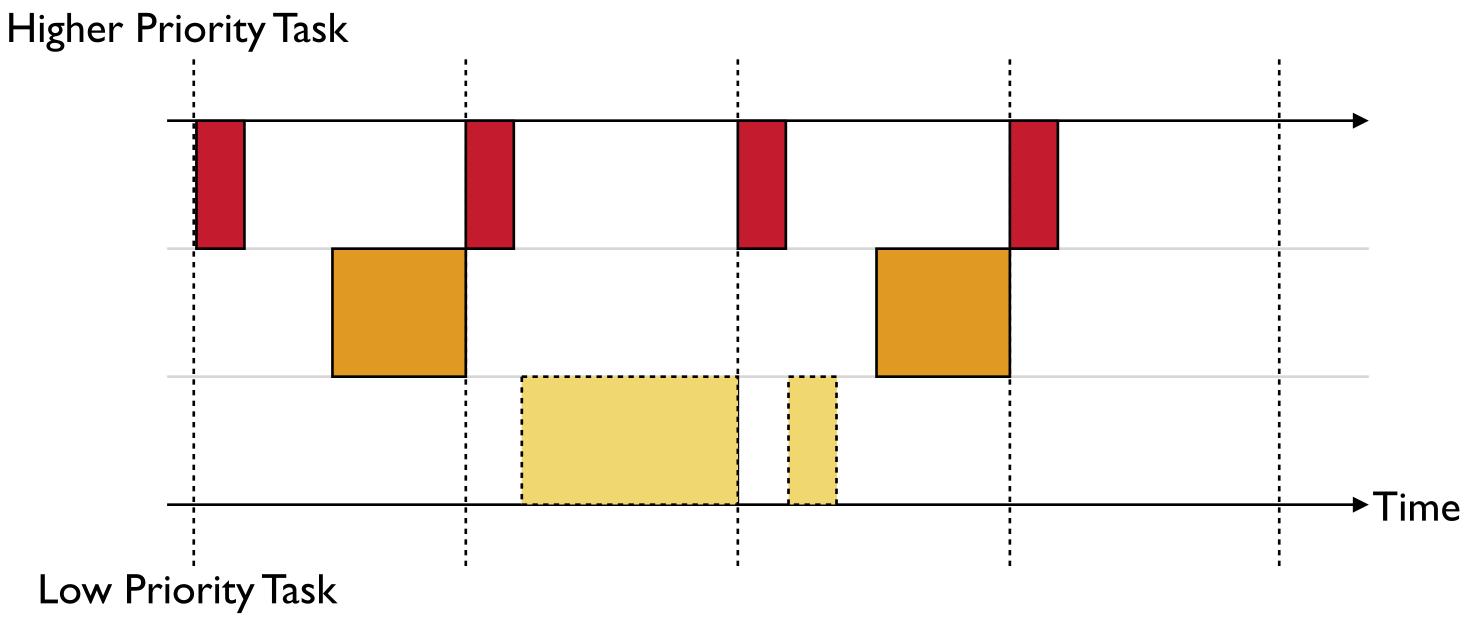

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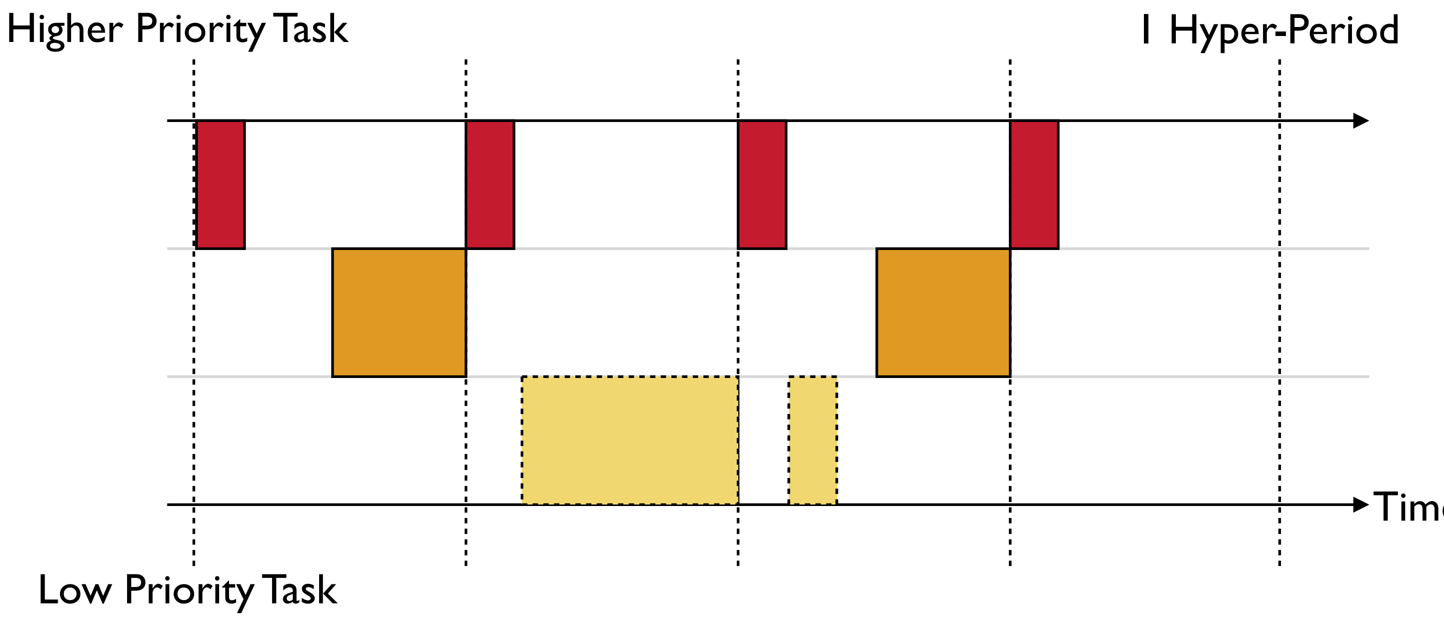
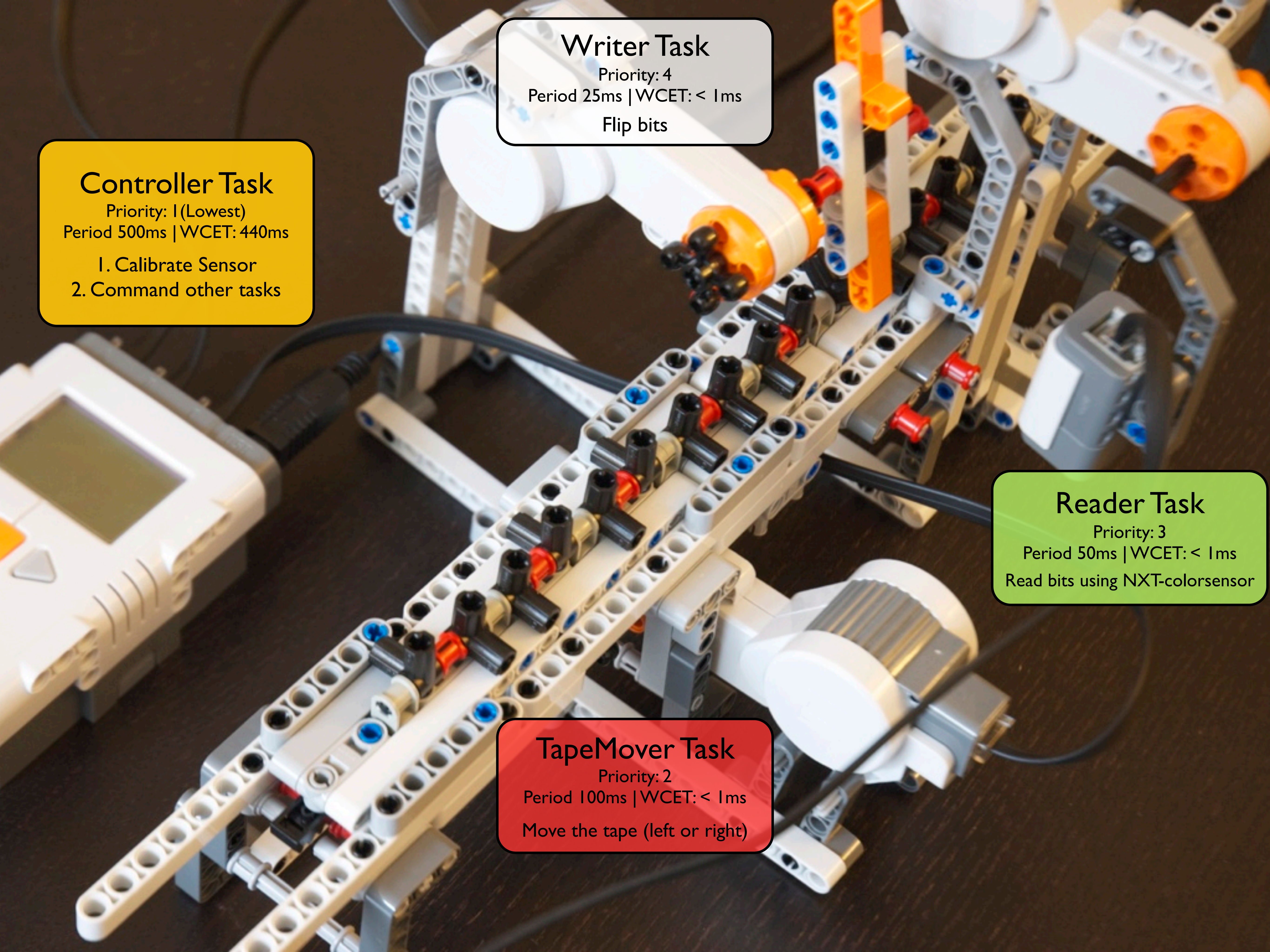


Preemptive Fixed Priority-based Scheduling

Case Study:

Concurrent Turing Machine



Writer Task
Priority: 4
Period 25ms | WCET: < 1ms
Flip bits
Controller Task
Priority: 1 (Lowest)
Period 500ms | WCET: 440ms
1. Calibrate Sensor
2. Command other tasks
Reader Task
Priority: 3
Period 50ms | WCET: < 1ms
Read bits using NXT-colorsensor
TapeMover Task
Priority: 2
Period 100ms | WCET: < 1ms
Move the tape (left or right)

Isn’t LEGO Mindstorms just a TOY?

SIEMENS

RENAULT

PSA PEUGEOT CITROËN

BMW

No, it runs OSEK/VDX-compatible RTOS.

Open Systems and their Interfaces for the Electronics in Motor Vehicles

a standard software architecture for the various electronic control units (ECUs) throughout a car

OPEL

BOSCH

DAIMLERCHRYSLER

DEMO

Unary Addition

2 + 3 = ?

http://www.youtube.com/watch?v=teDyd0d5M4o

Properties

Property 1: When a bit is being read, all the motors should **stop**.

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Property 2: When writer flips a bit, the tape motor and read motor should **stop**.

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Property 3: When tape moves, the writer motor and read motor should **stop**.

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Property 4: When a bit is being read, the sensor should be on **Green** mode

Properties

Property 1: When a bit is being read, all the motors should **stop**.

Property 2: When writer flips a bit, the tape motor and read motor should **stop**.

Property 3: When tape moves, the writer motor and read motor should **stop**.

Property 4: When a bit is being read, the sensor should be on **Green** mode

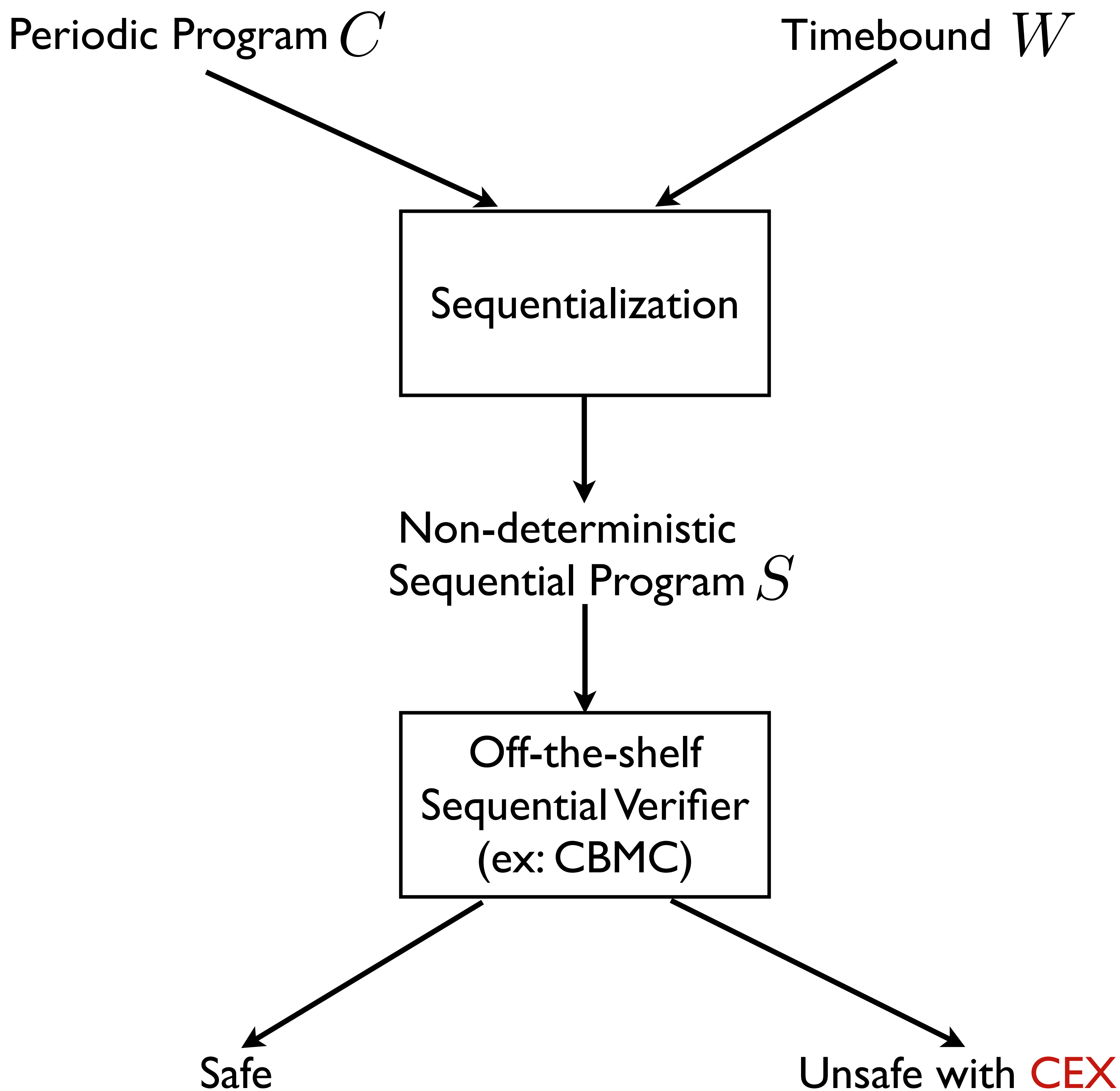
Property 5: The sensor mode must be switched in **Controller Task**, not in Reader Task

Properties

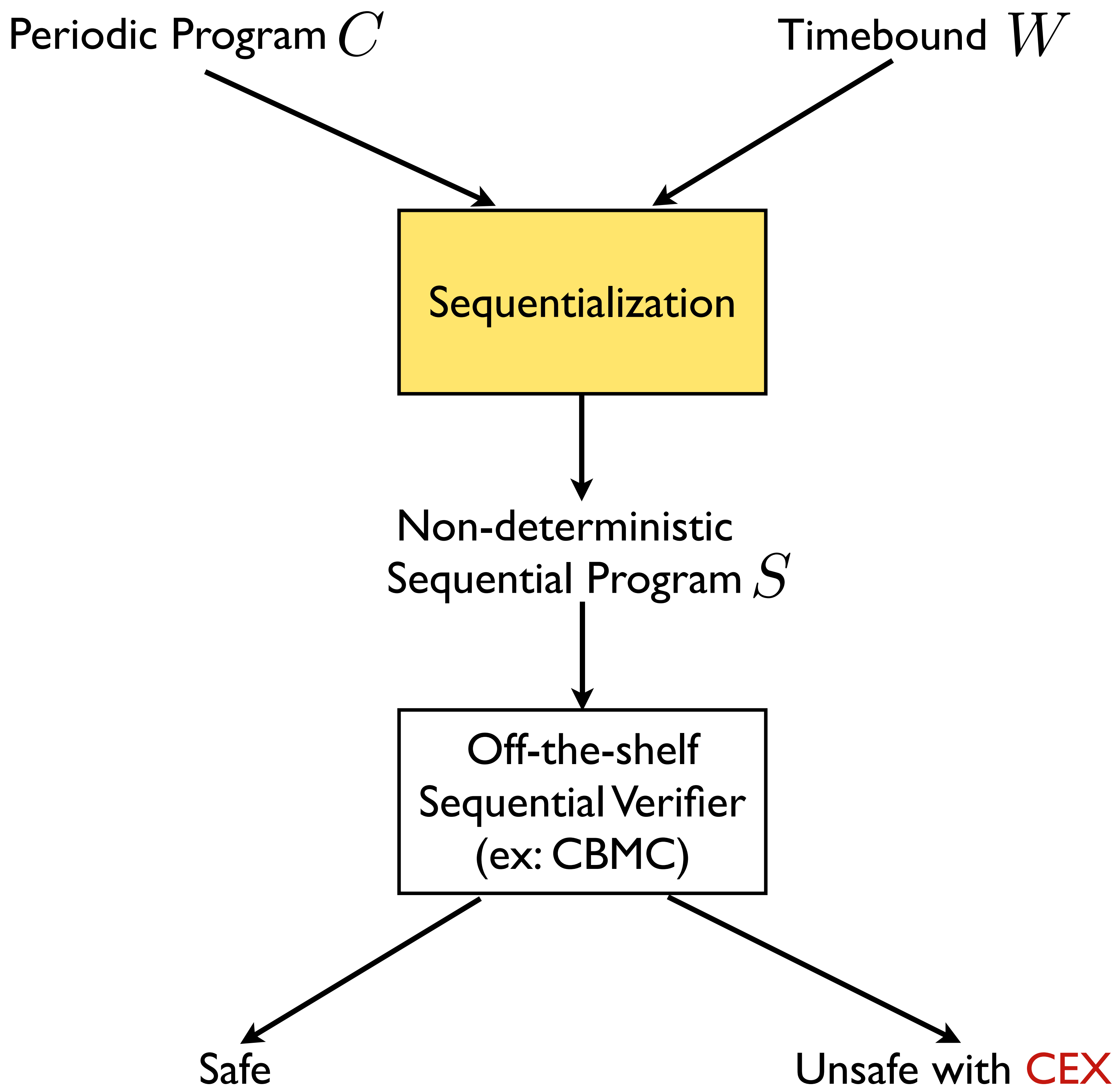
```
  case READ_SENSOR:
    if(ecrobot_get_nxtcolorsensor_mode(COLOR_SENSOR) != NXT_LIGHTSENSOR_GREEN) {
      ecrobot_set_nxtcolorsensor(COLOR_SENSOR, NXT_LIGHTSENSOR_GREEN);
      W(need_to_run_nxtbg, true);
    }

    if(!R(need_to_run_nxtbg)) {
      /* Turn the sensor on */
#ifdef VERIFICATION
      /* Property 1: When a bit is being read,
         all the motors should be stopped.  */
      /* PASSED with 80*/
      assert(R(R_speed) == 0 && R(W_speed) == 0 && R(T_speed) == 0);

      /* Property 4: When a bit is being read,
         the sensor should be on Green mode */
      assert(ecrobot_get_nxtcolorsensor_mode(COLOR_SENSOR) == NXT_LIGHTSENSOR_GREEN);
#endif
      ecrobot_set_nxtcolorsensor(COLOR_SENSOR, NXT_LIGHTSENSOR_GREEN);

      /* Read Sensor Value */
      bg_nxtcolorsensor(false);
      color = ecrobot_get_nxtcolorsensor_light(COLOR_SENSOR);
```

Key Idea:

Sequentialization



Time-bounded Verification of Periodic Programs via Sequentialization



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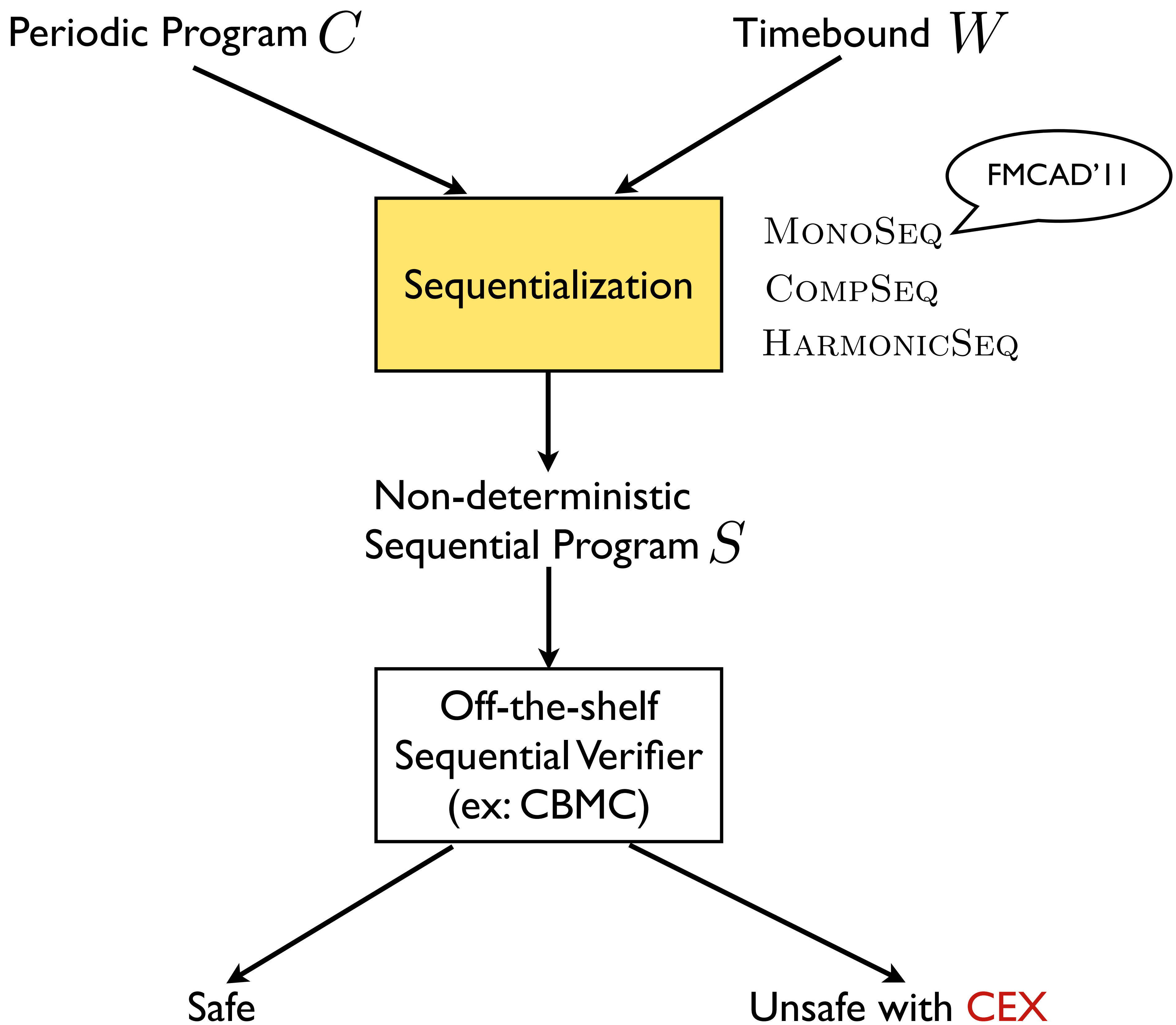

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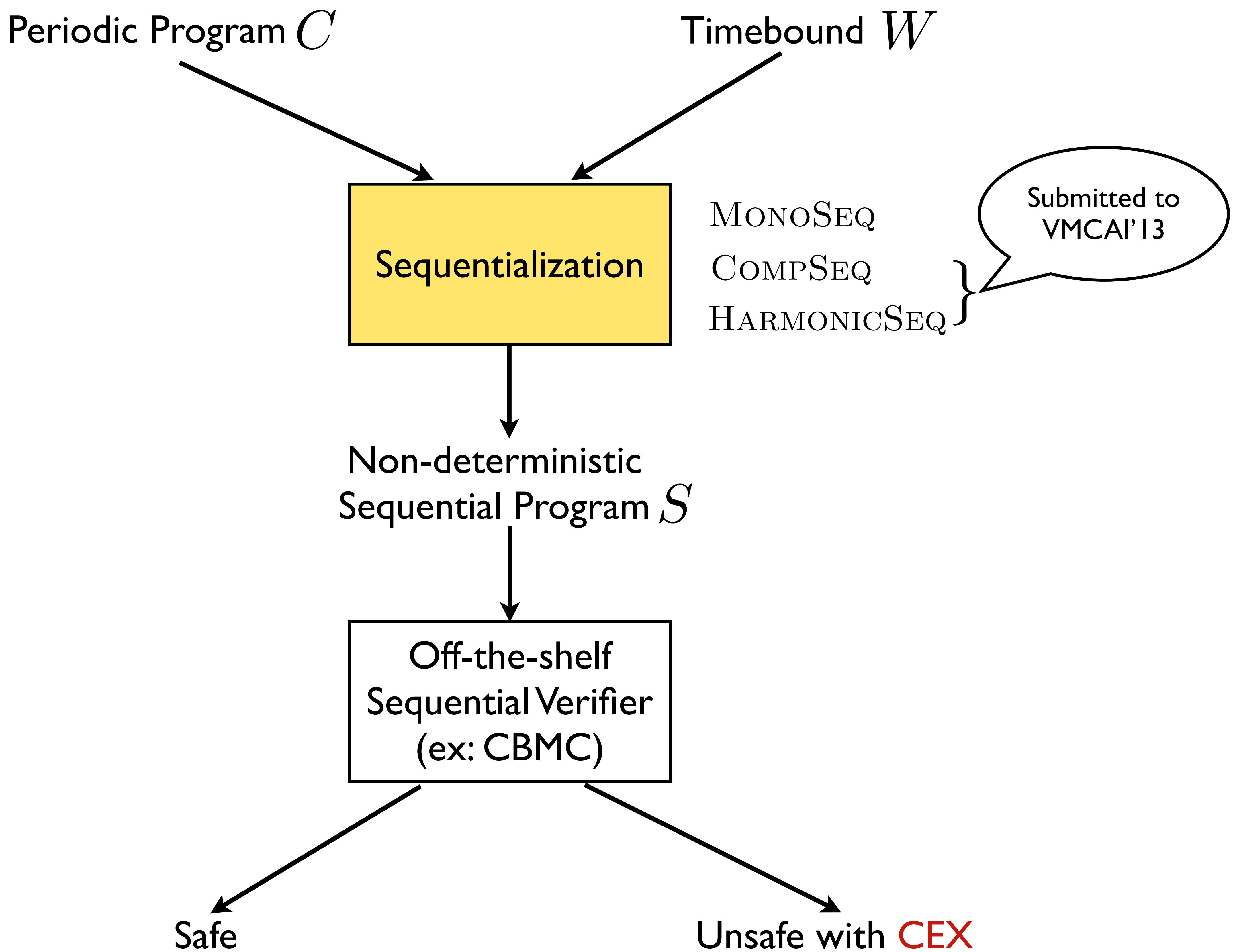

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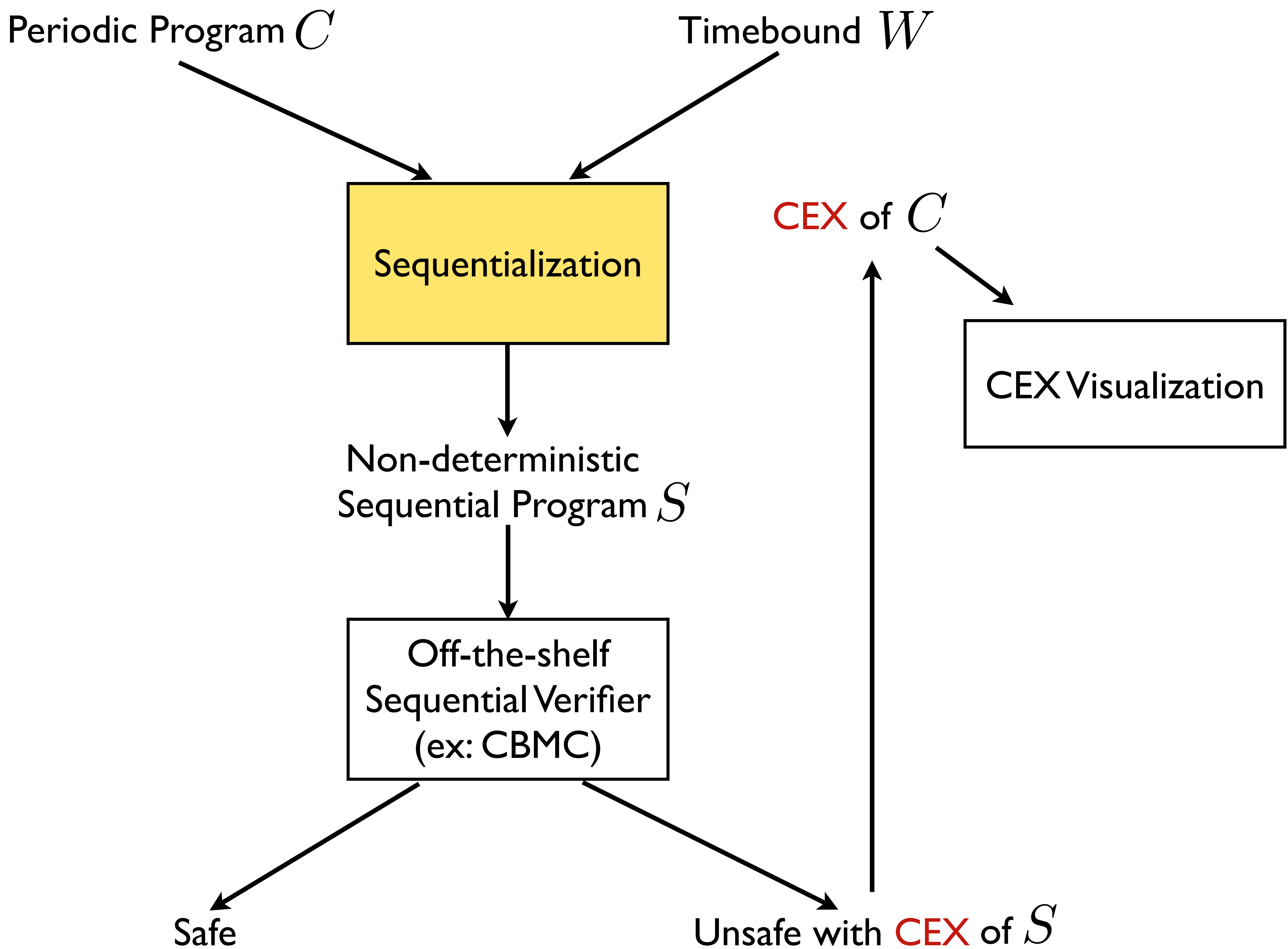

Time-bounded Verification of Periodic Programs via Sequentialization

Naive Approach:

1. Enumerate all possible (sequentialized) executions
2. Verify each of them

MonoSeq Sequentialization

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Exponential Blow-up!

MONOSEQ Sequentialization

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Our Approach (MonoSeq):

1. Construct a **non-deterministic** sequentialized program
2. Enforce legal job scheduling and prune out infeasible thread executions by adding **constraints**

MonoSeq Sequentialization

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Program + Constraints

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MonoSeq Sequentialization

Naive Approach:

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2. Verify each of them

Ex:

```
x:= nondet();
y:= 10;
assume(x > 10);
```

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MonoSeq Sequentialization

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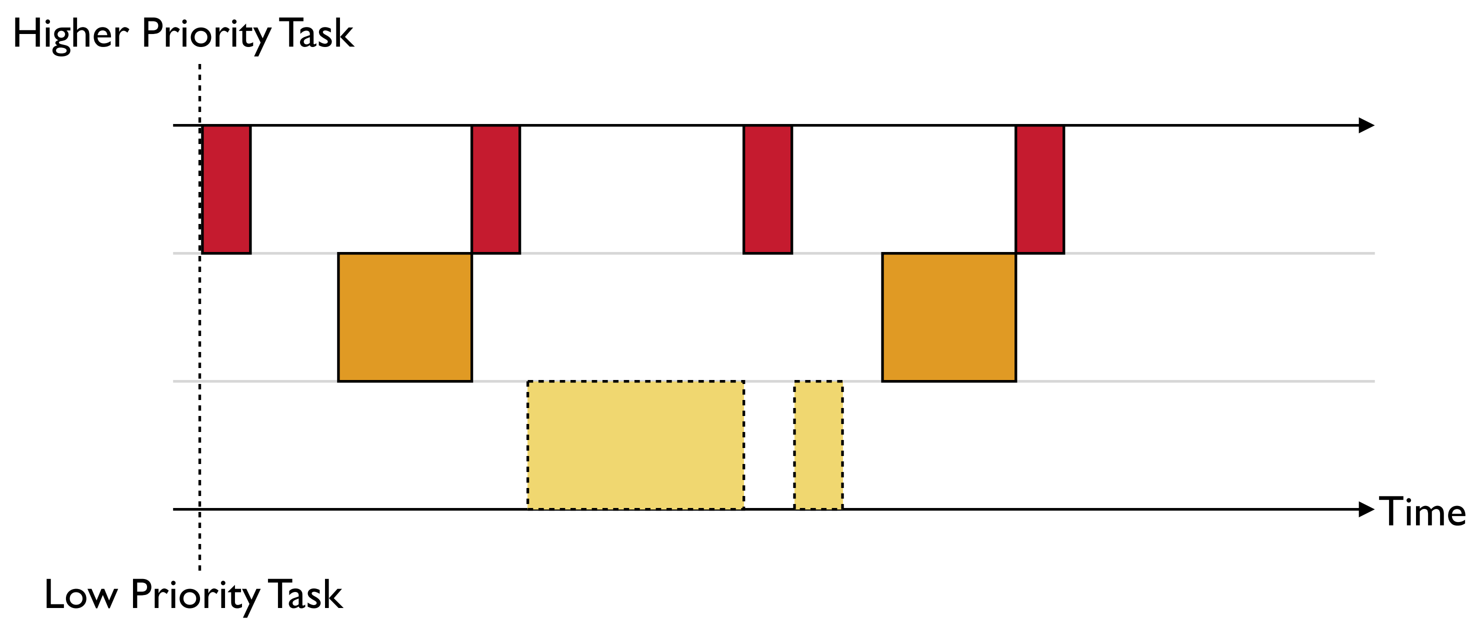
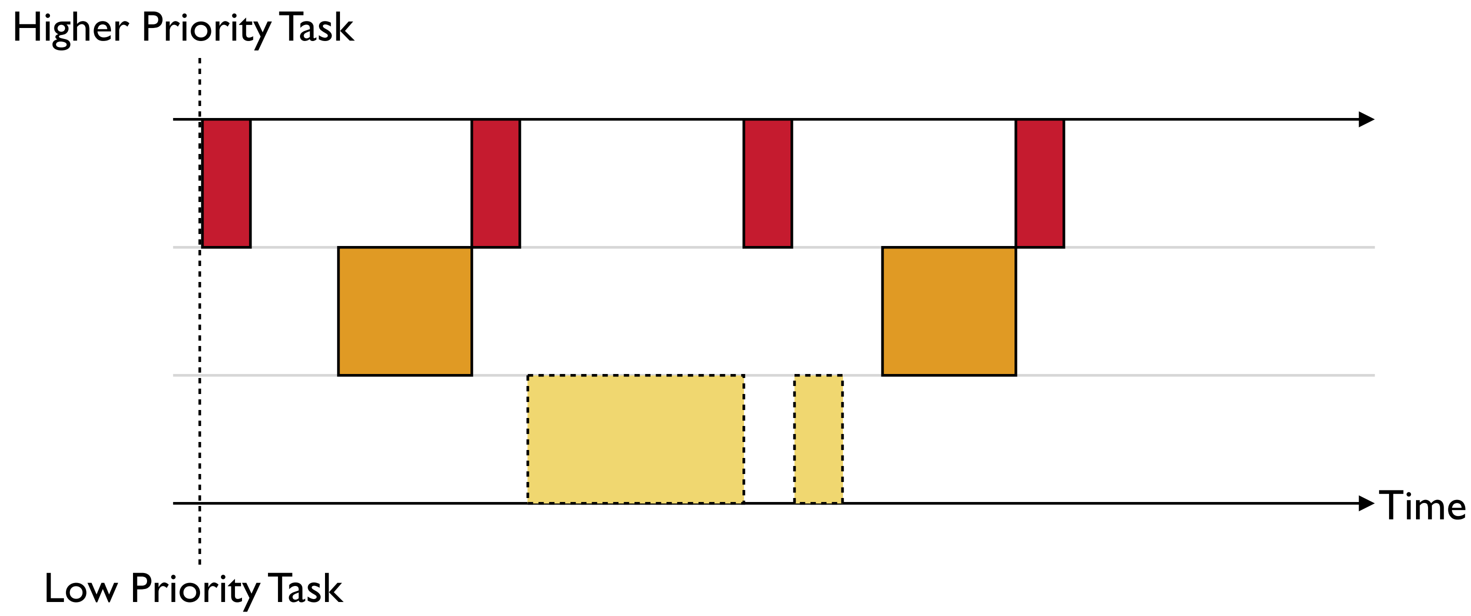
$O(R)$
where R= # of Jobs

$O(R^2)$
where R= # of Jobs

MonoSeq Sequentialization



Job-bounded Abstraction



Because we are interested in logical properties,
We abstract the absolute time with relative order of execution

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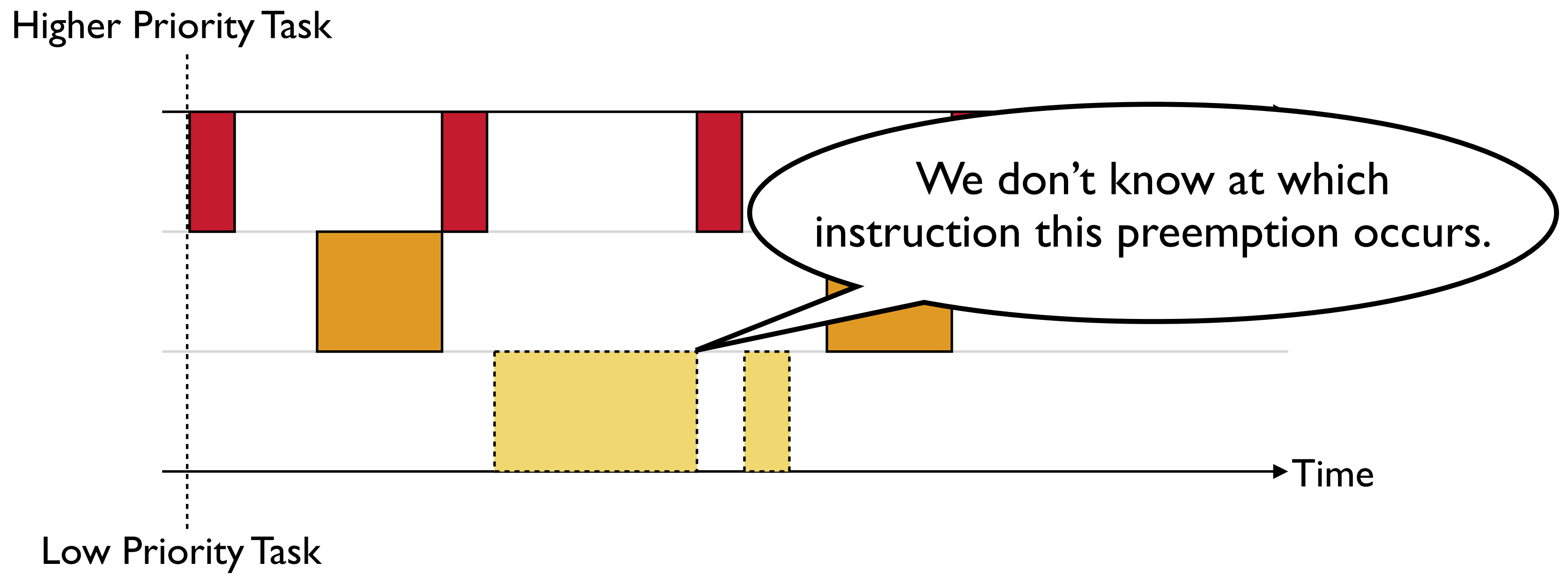

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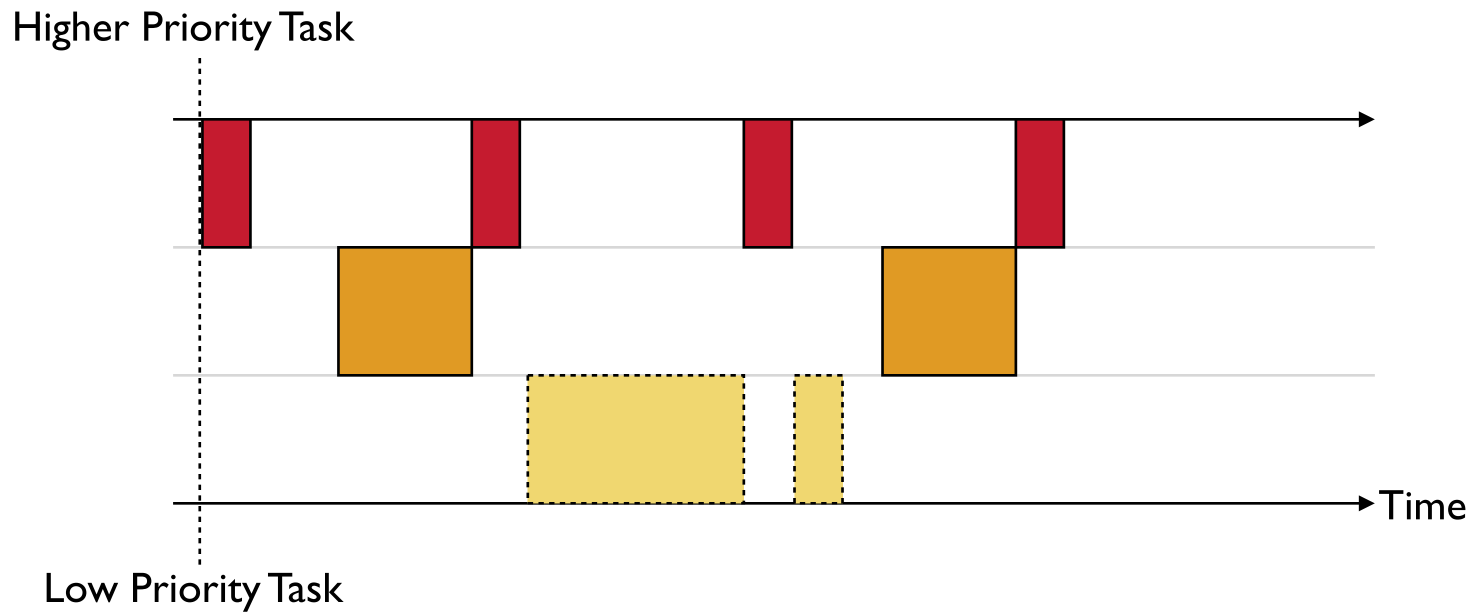

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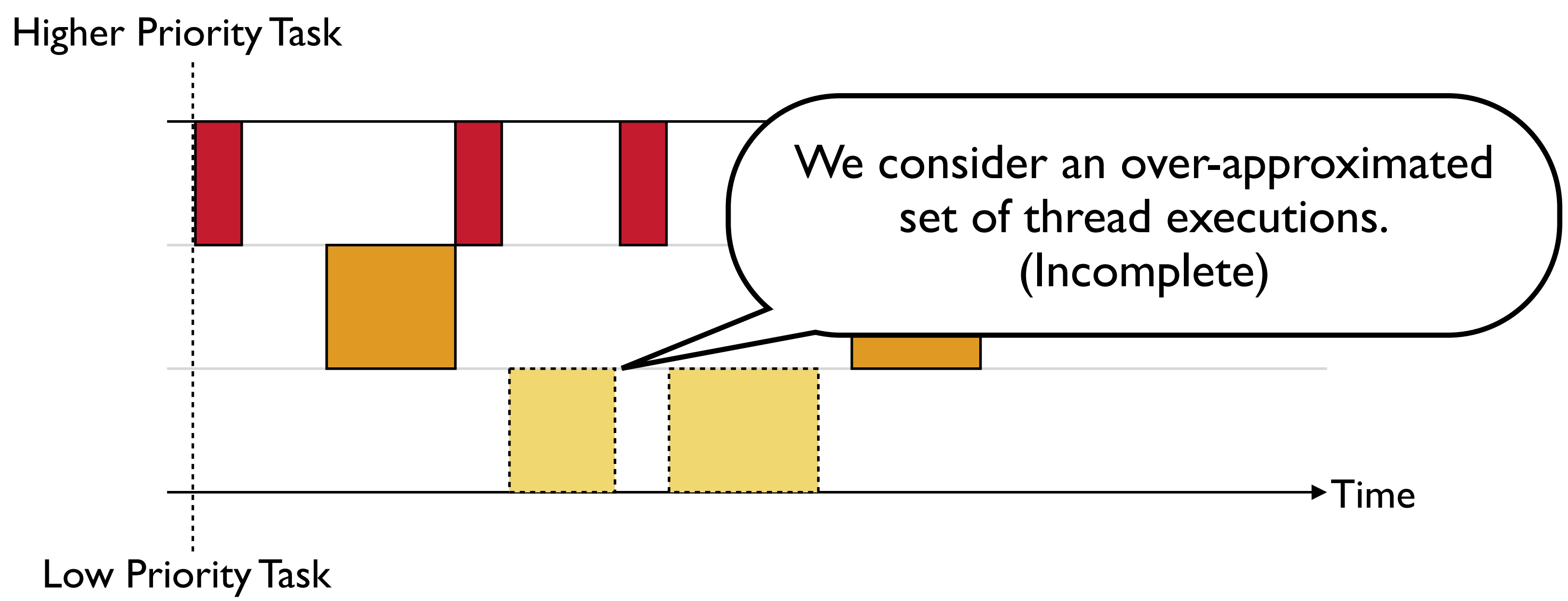

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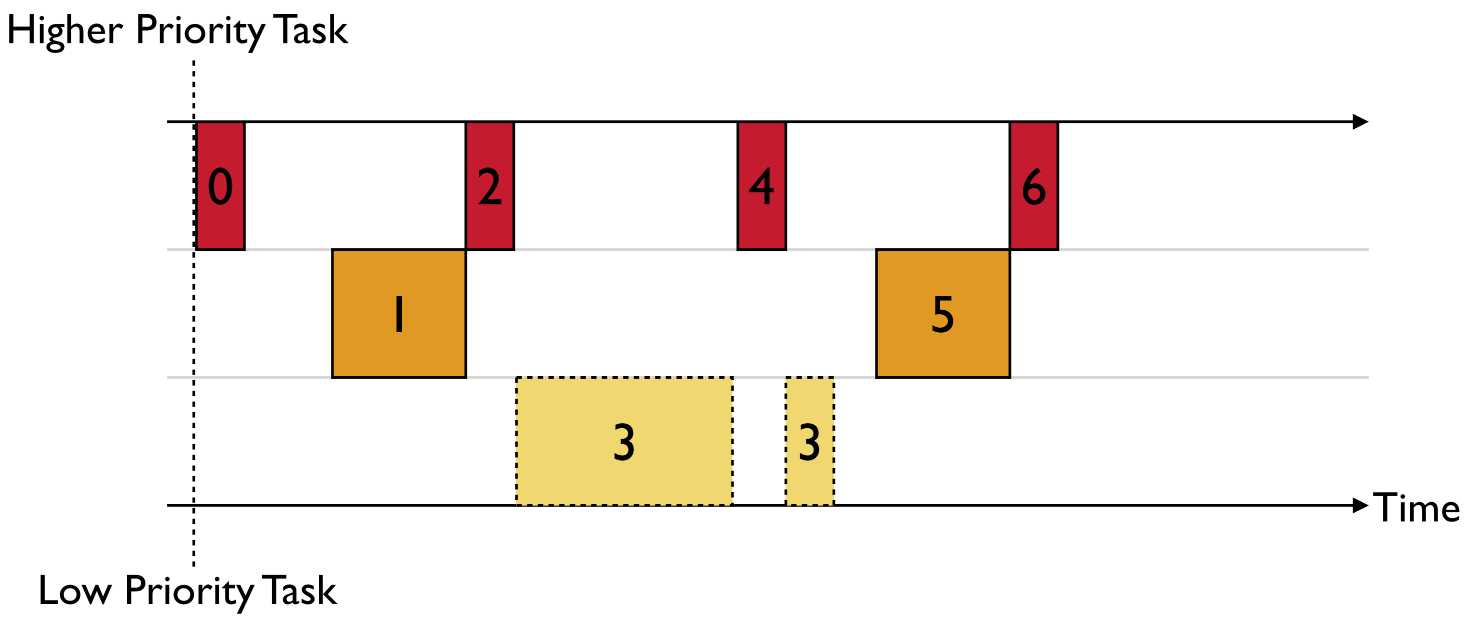
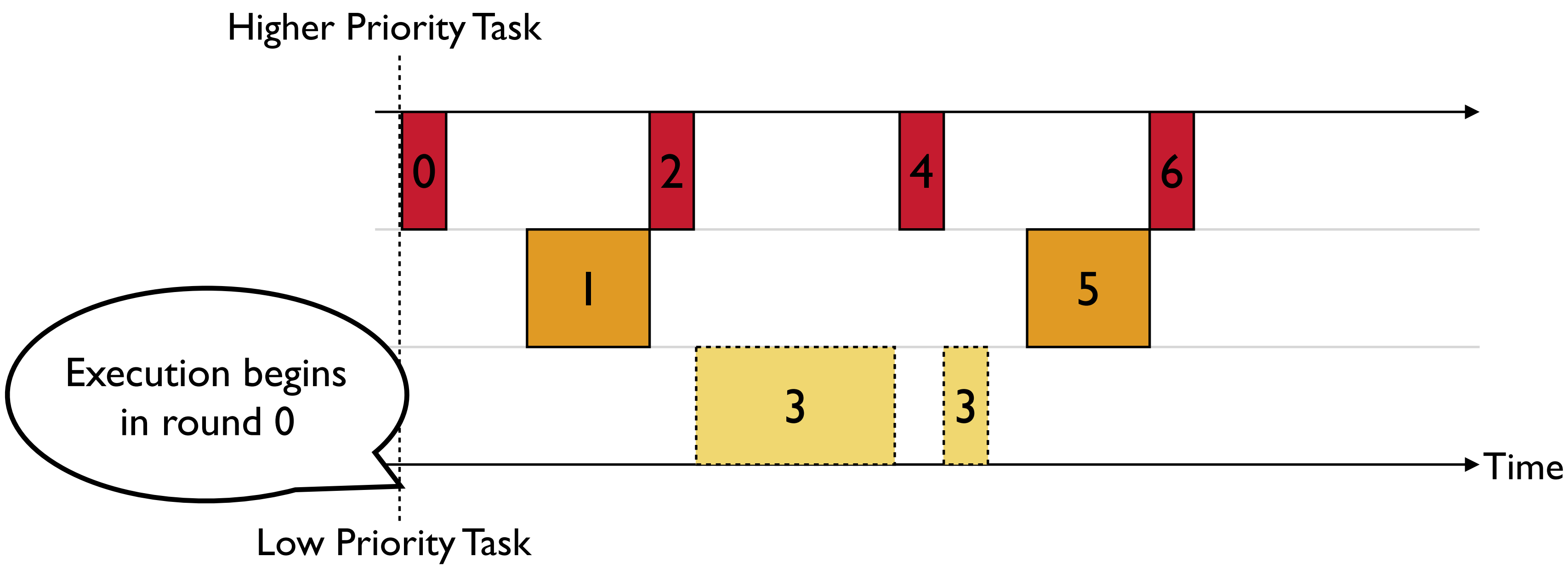


Observation:
Any execution can be partitioned into *scheduling rounds*

Partition Execution into Rounds



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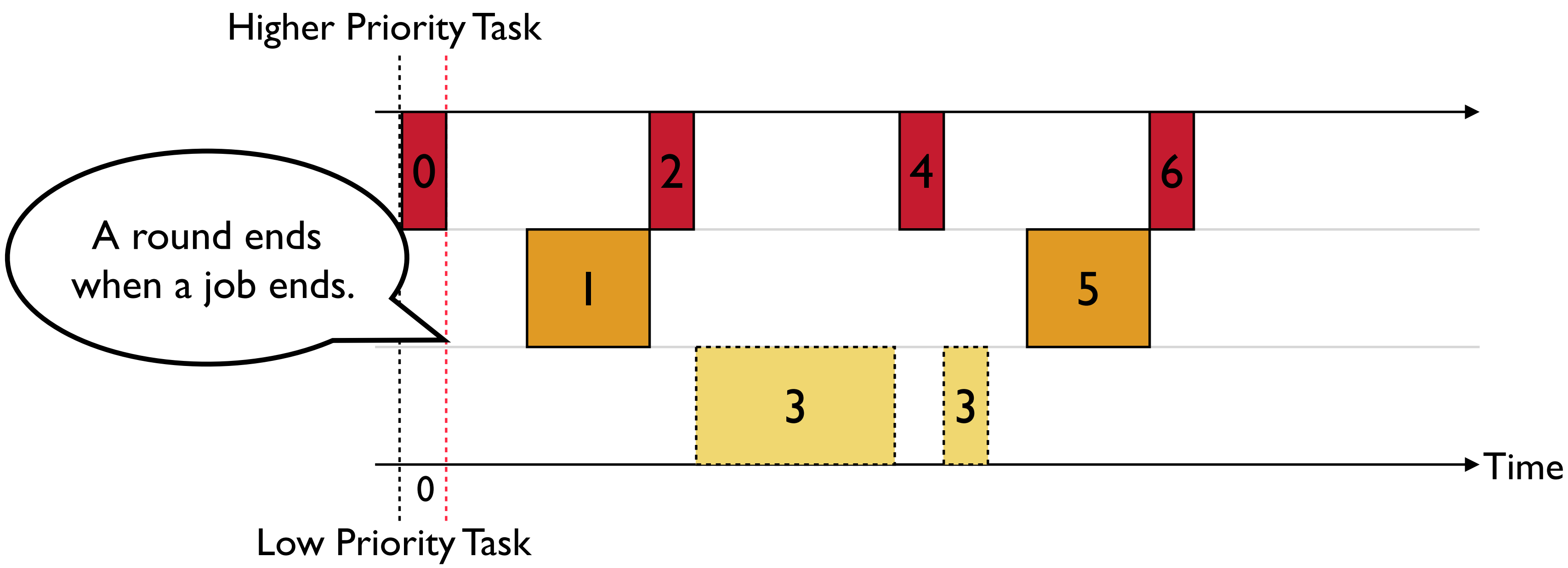

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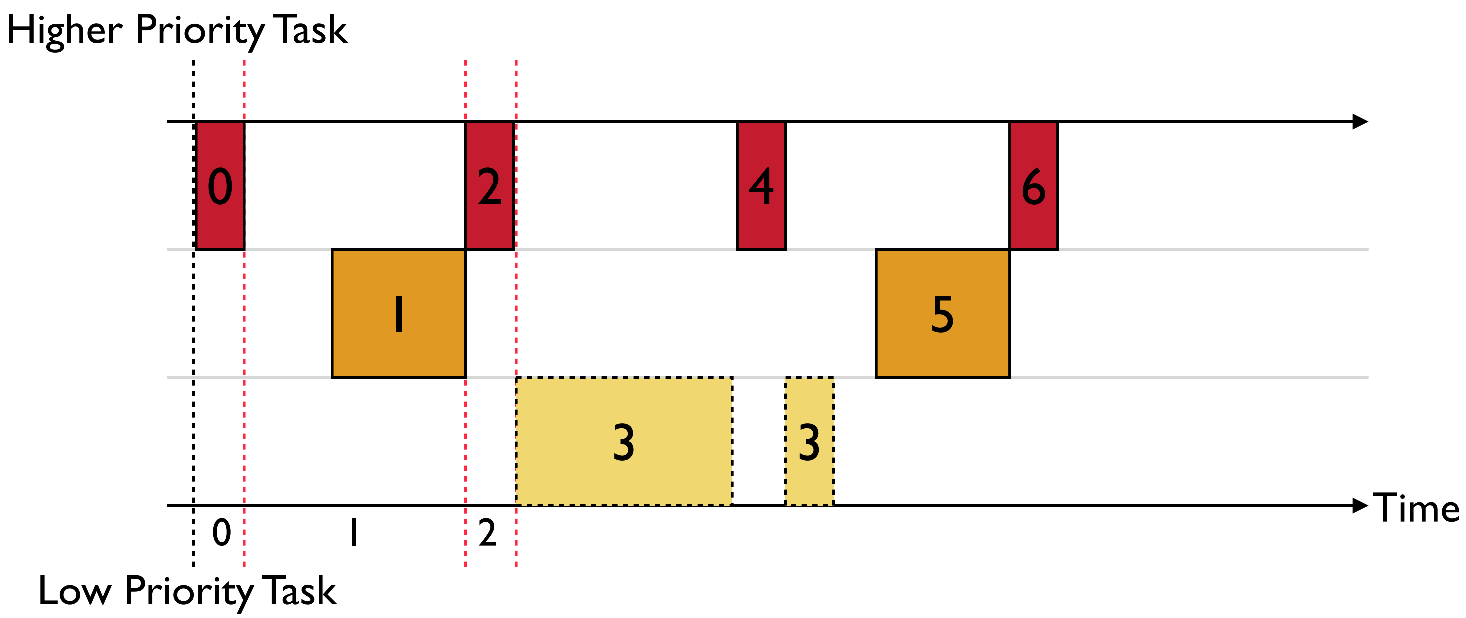

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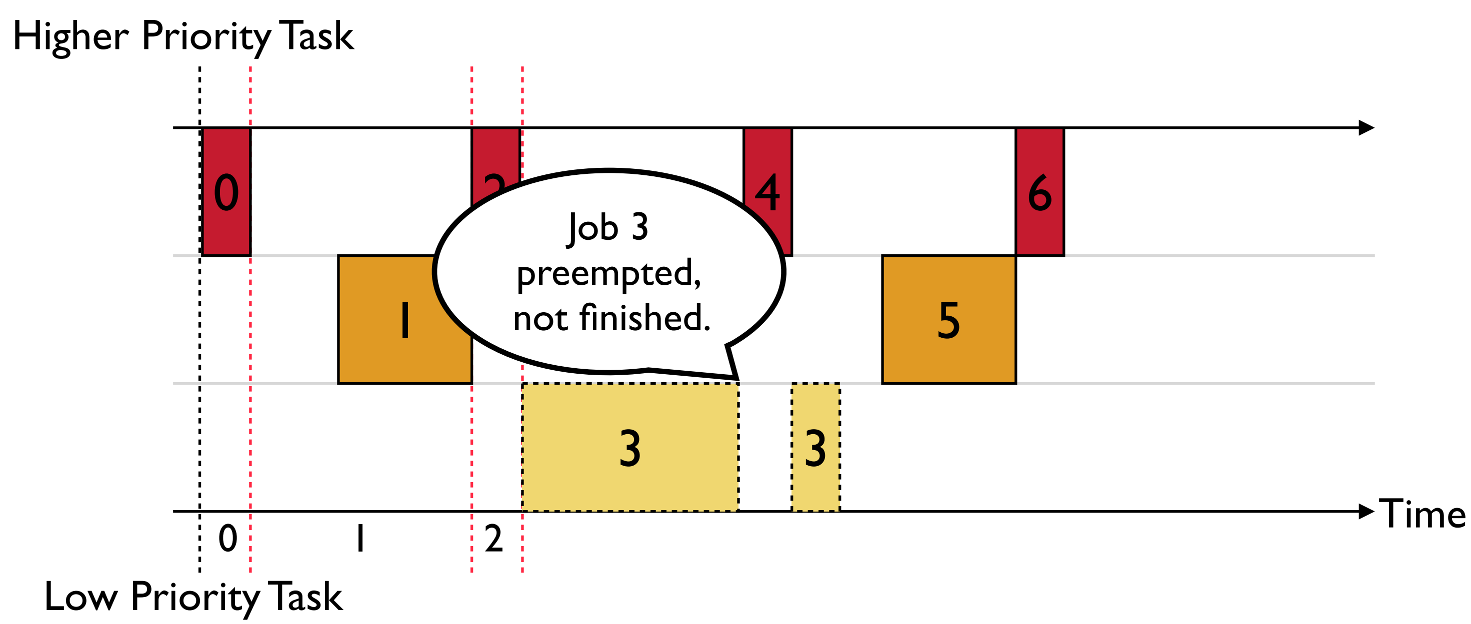

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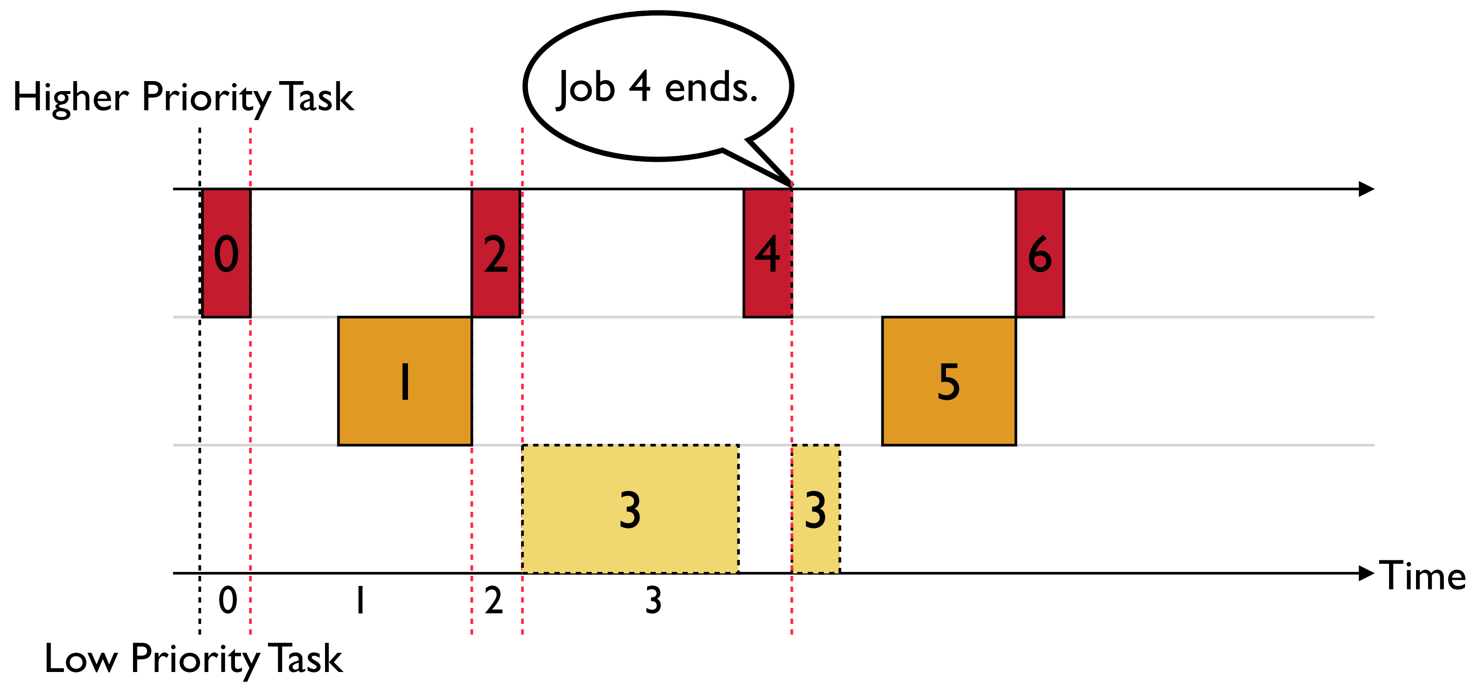

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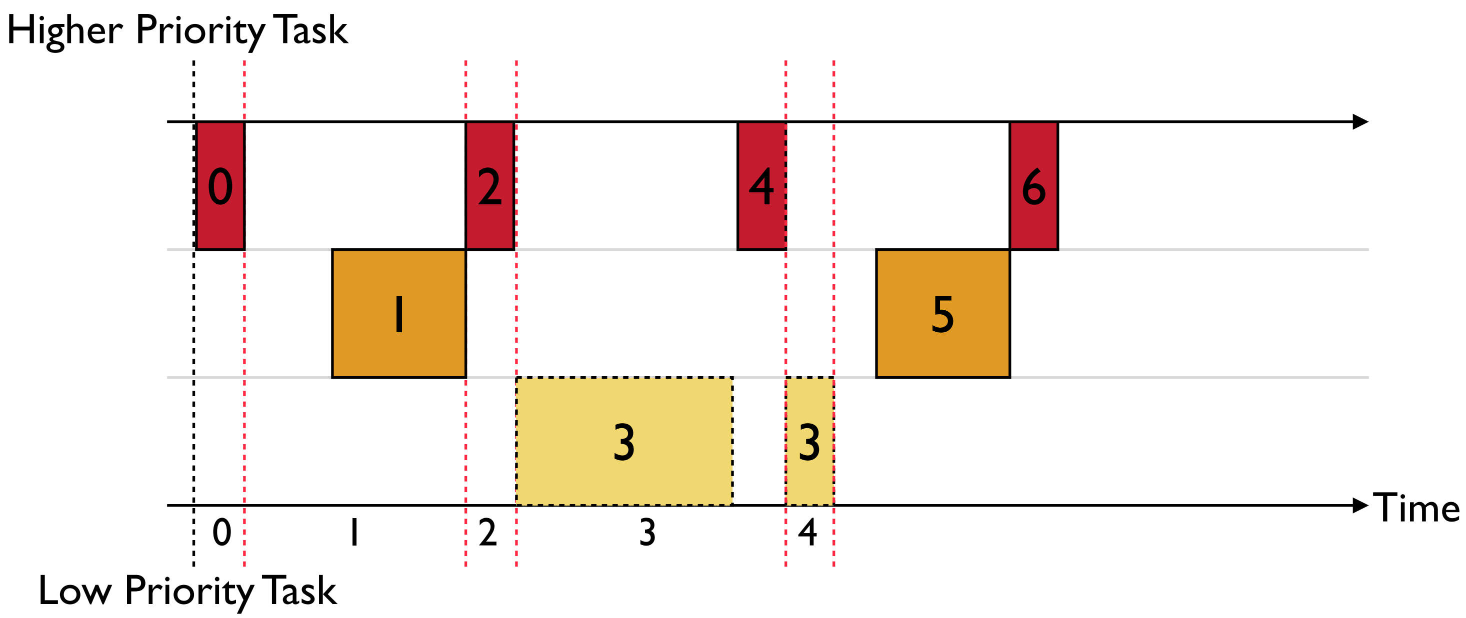

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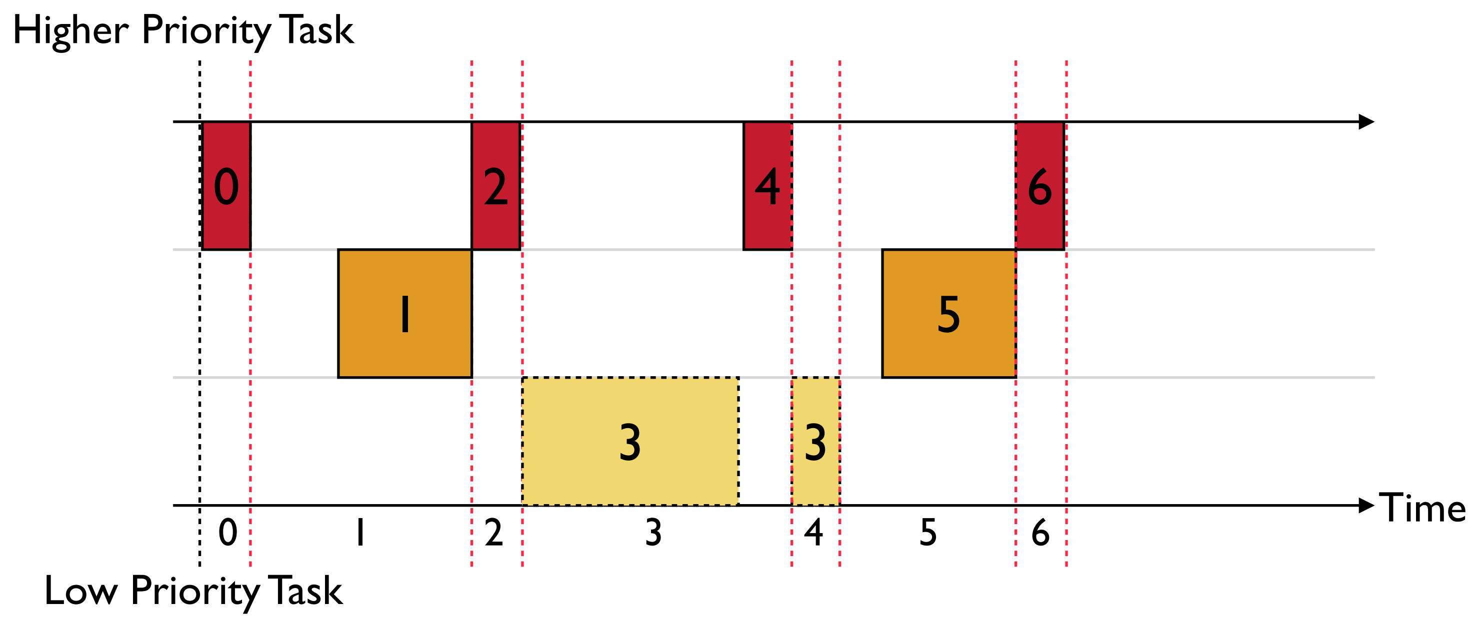

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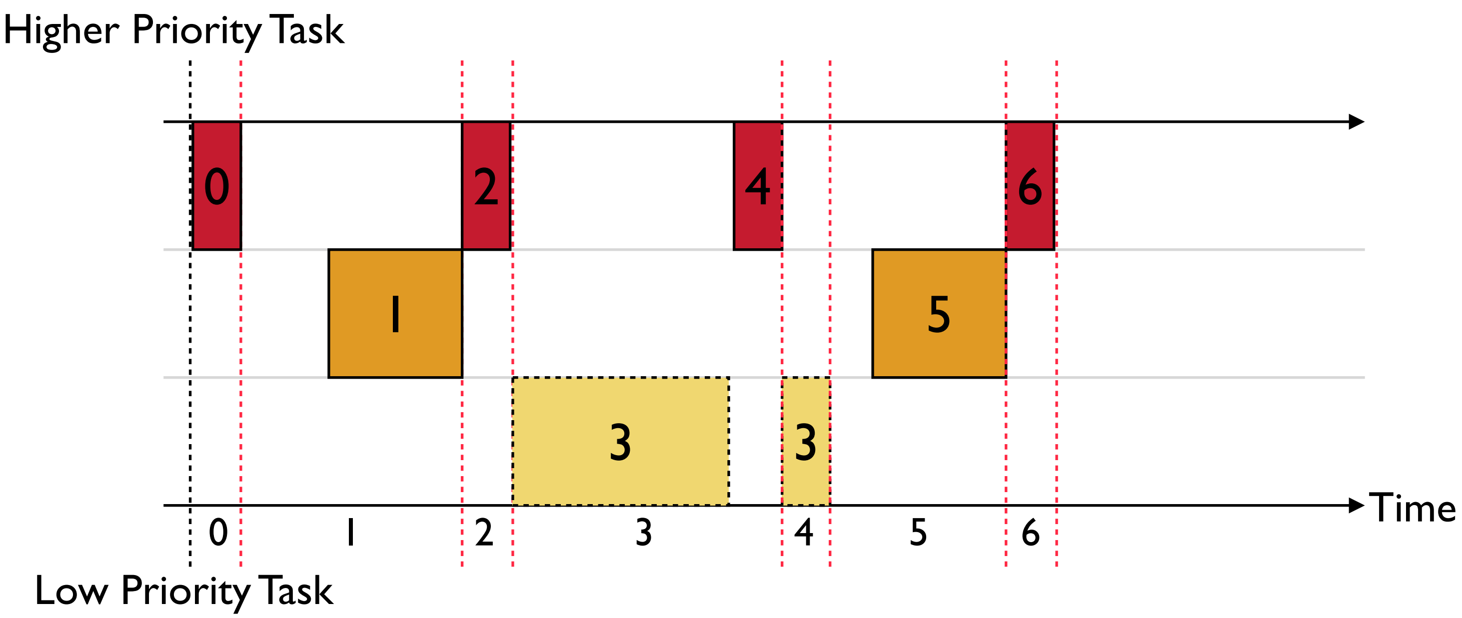

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$$\# \text{ of Jobs} = \# \text{ of Rounds}$$

Partition Execution into Rounds

Add constraints to enforce **legal** job scheduling

1. Jobs are sequential:

MonoSeq Sequentialization

Add constraints to enforce **legal** job scheduling

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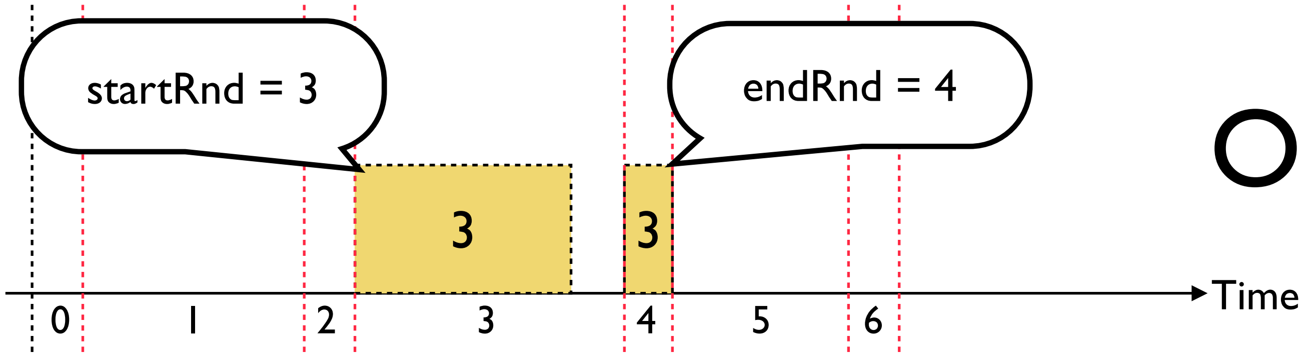

MONOSEQ Sequentialization

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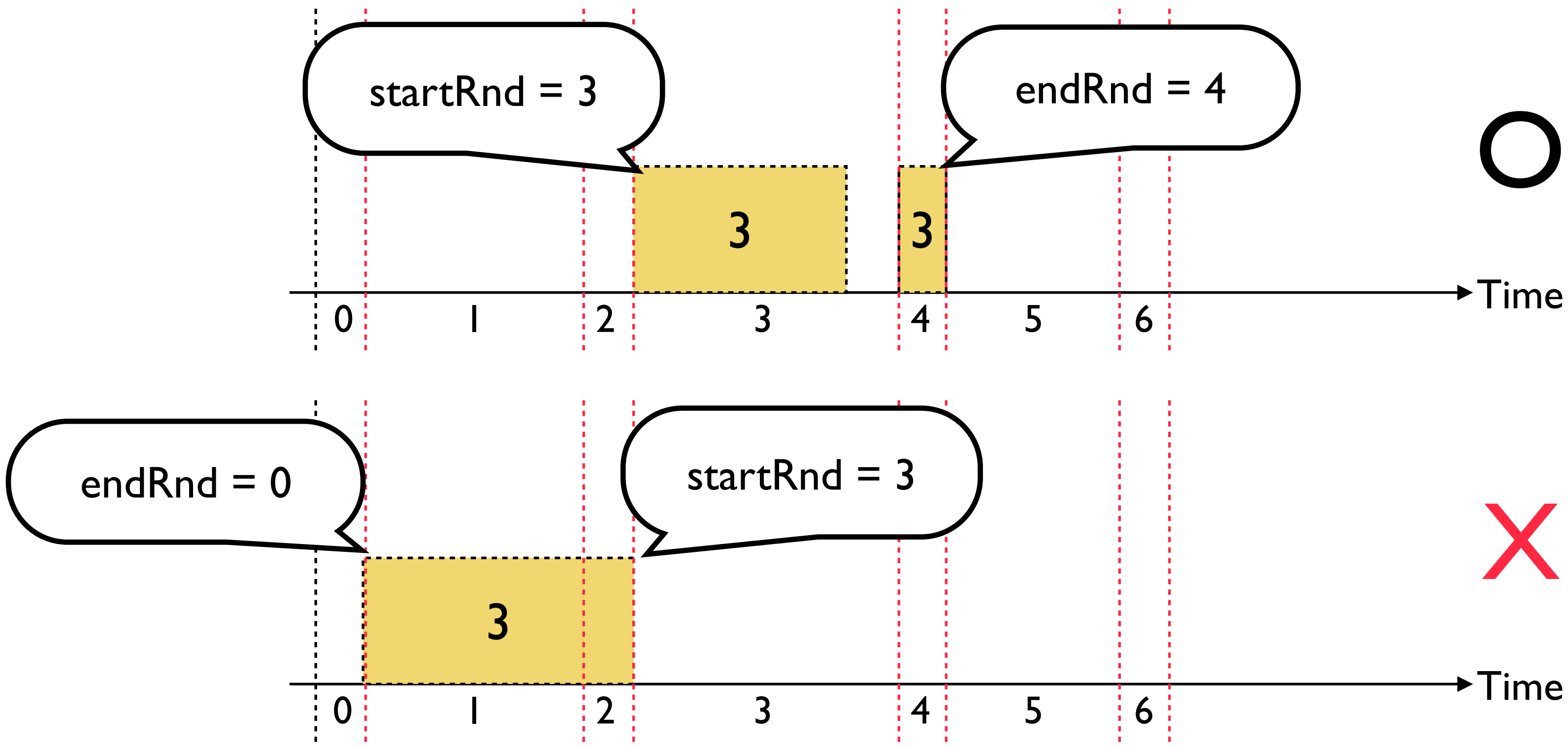

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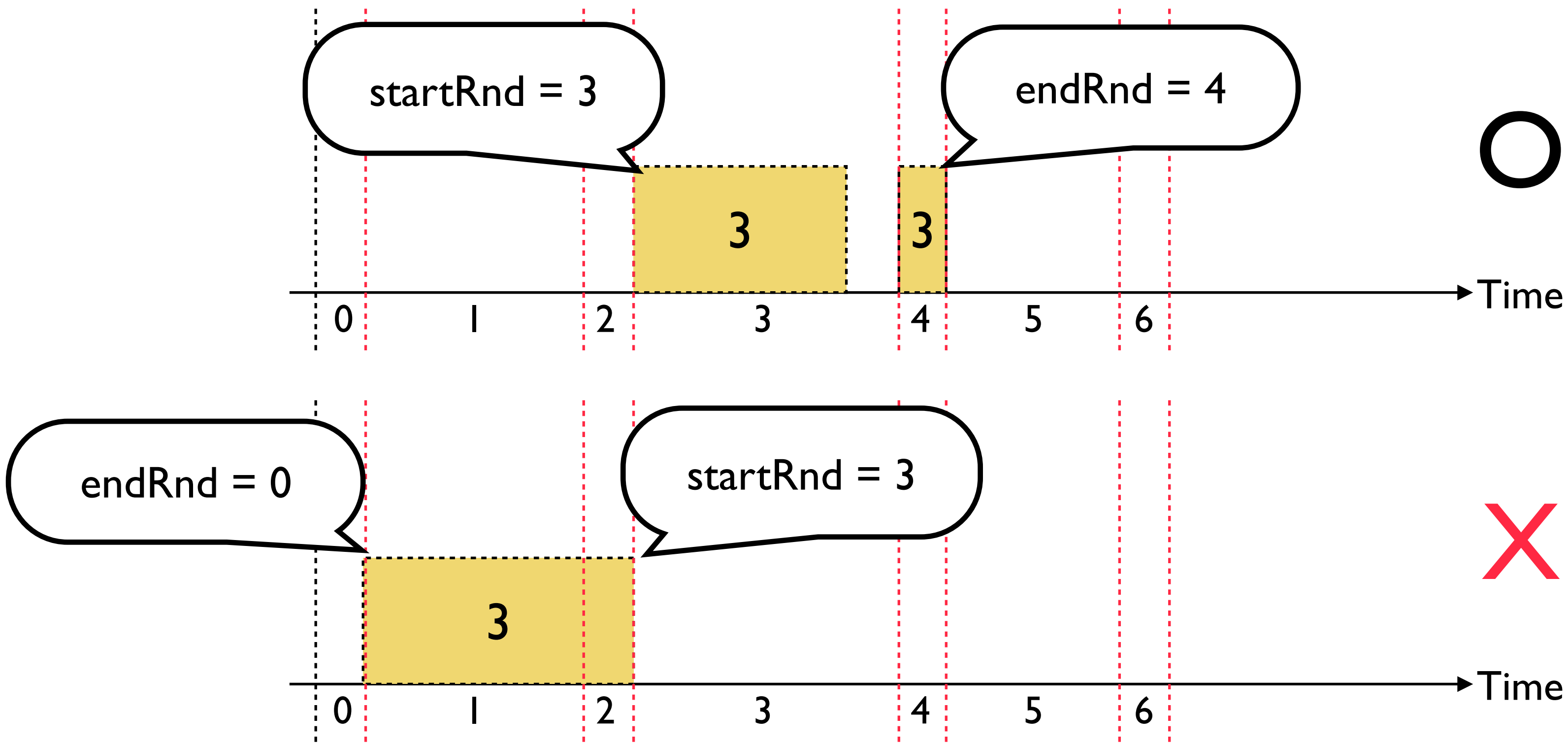

// Jobs are sequential

$$\forall t \in \mathsf{T}, j \in \mathsf{J}(t) \cdot$$
$$\text{assume}($$
$$0 \leq start[t][j] \leq end[t][j] \leq R \wedge$$
$$(\neg \text{last}(t, j) \Rightarrow end[t][j] \leq start[t][j+1]))$$

MonoSeq **Sequentialization**

Add constraints to enforce **legal** job scheduling

2. Jobs are well-nested:

MonoSeq Sequentialization

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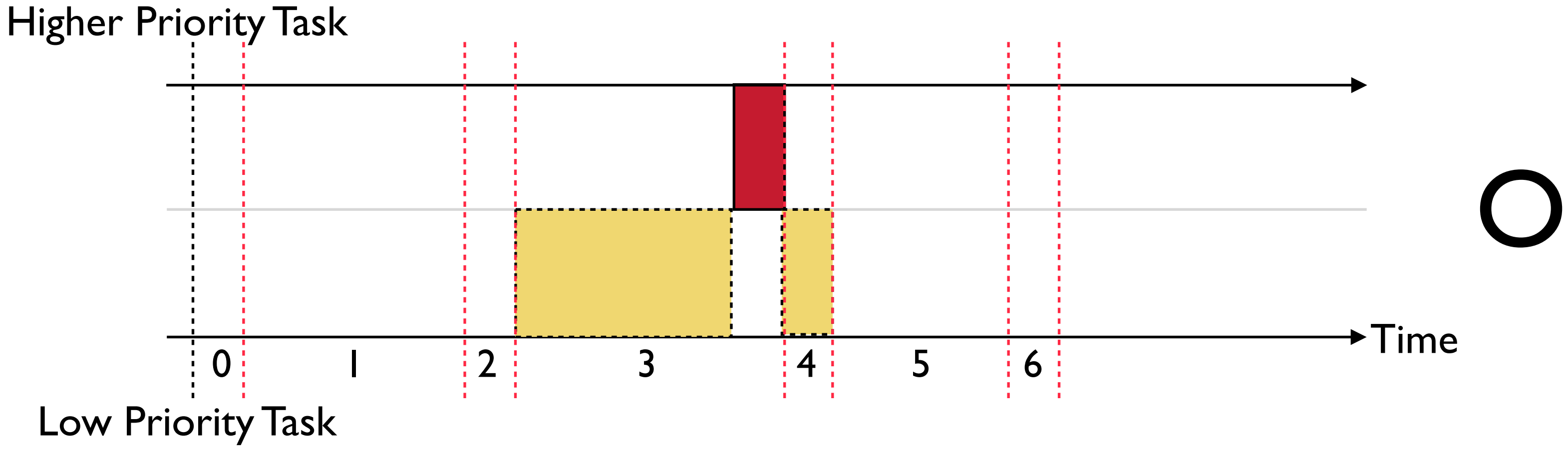

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Add constraints to enforce **legal** job scheduling

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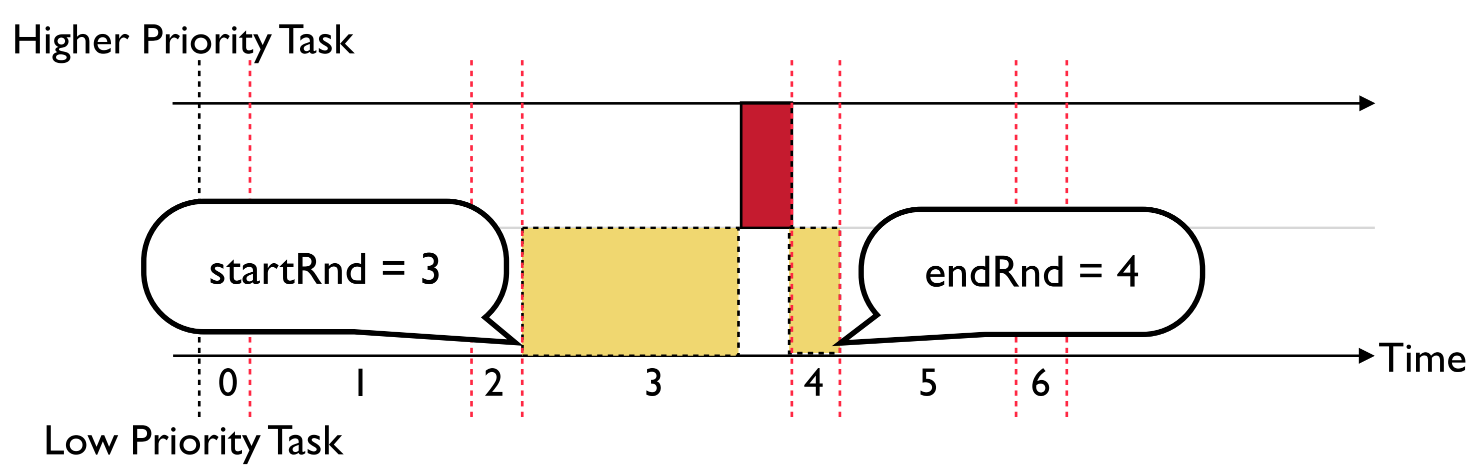

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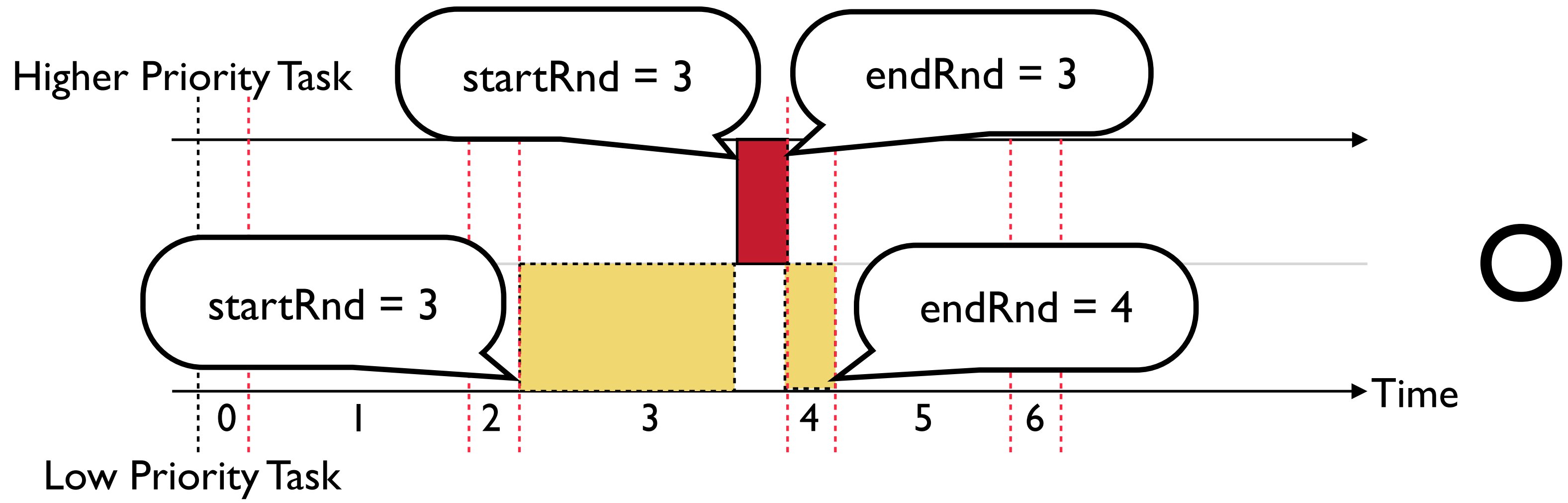

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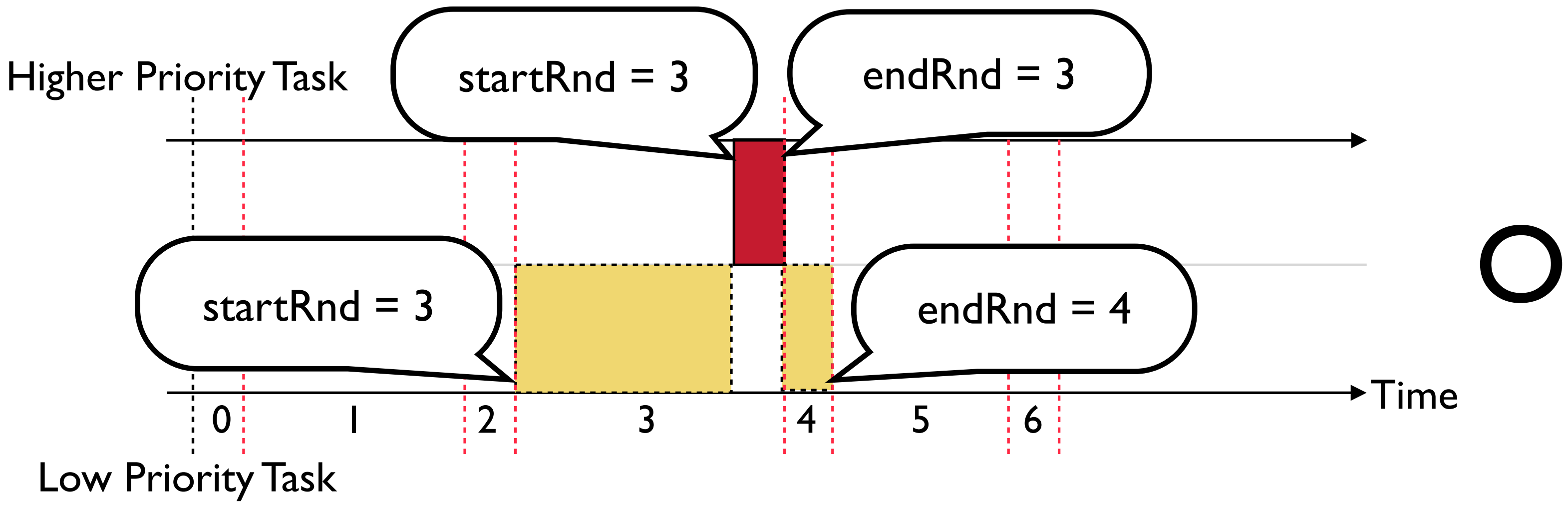

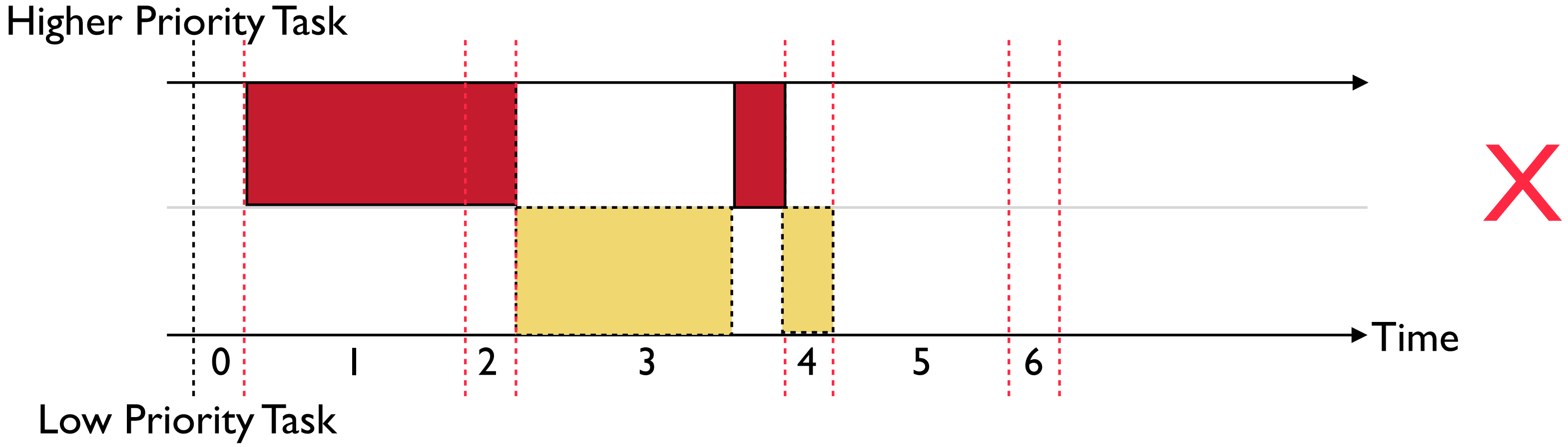

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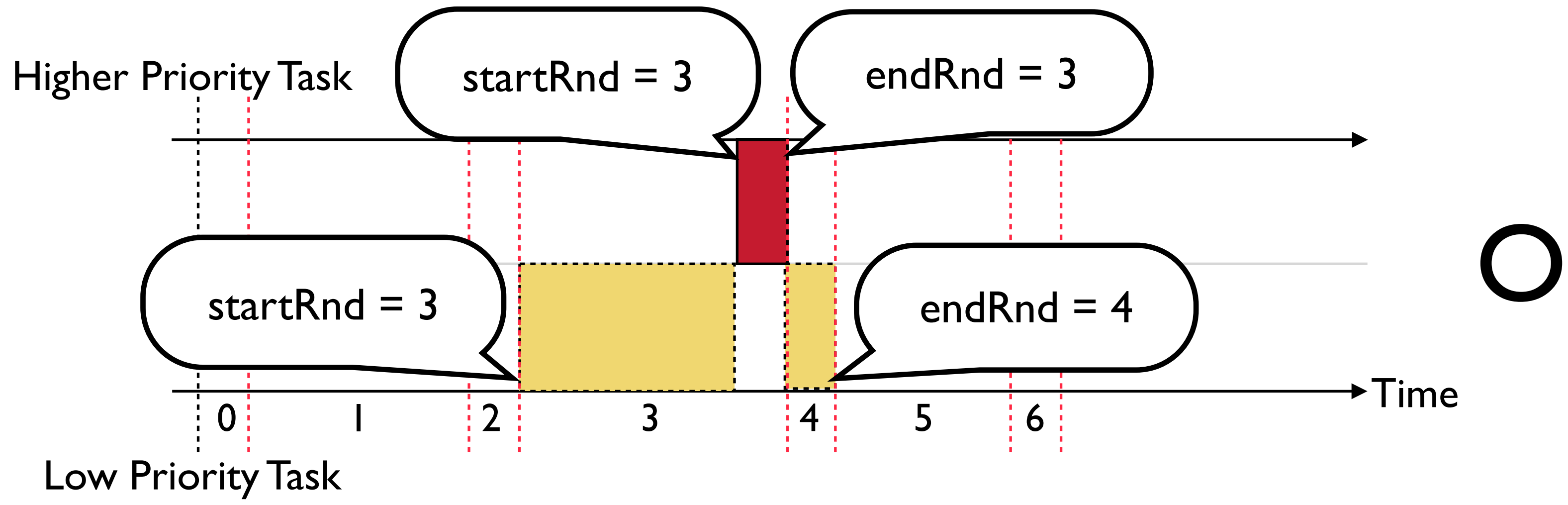
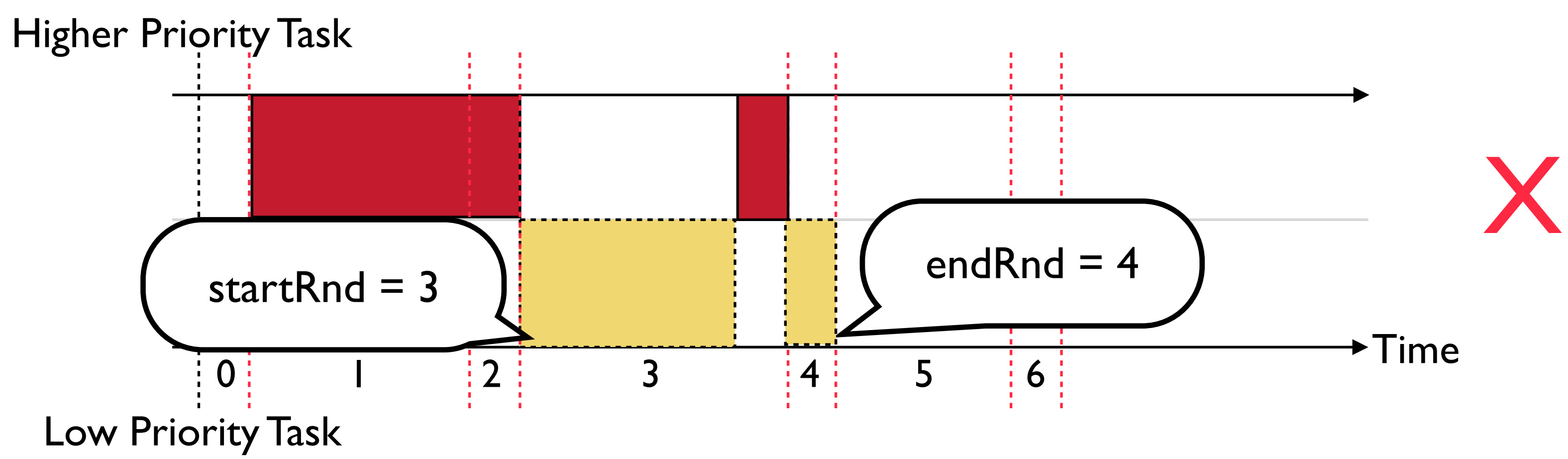
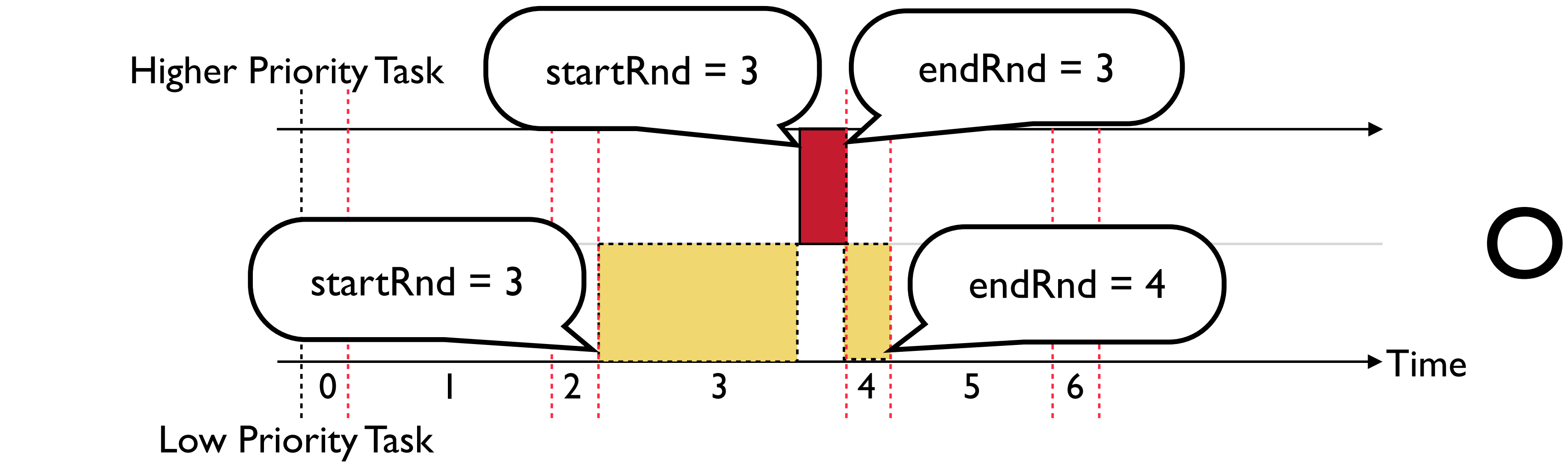


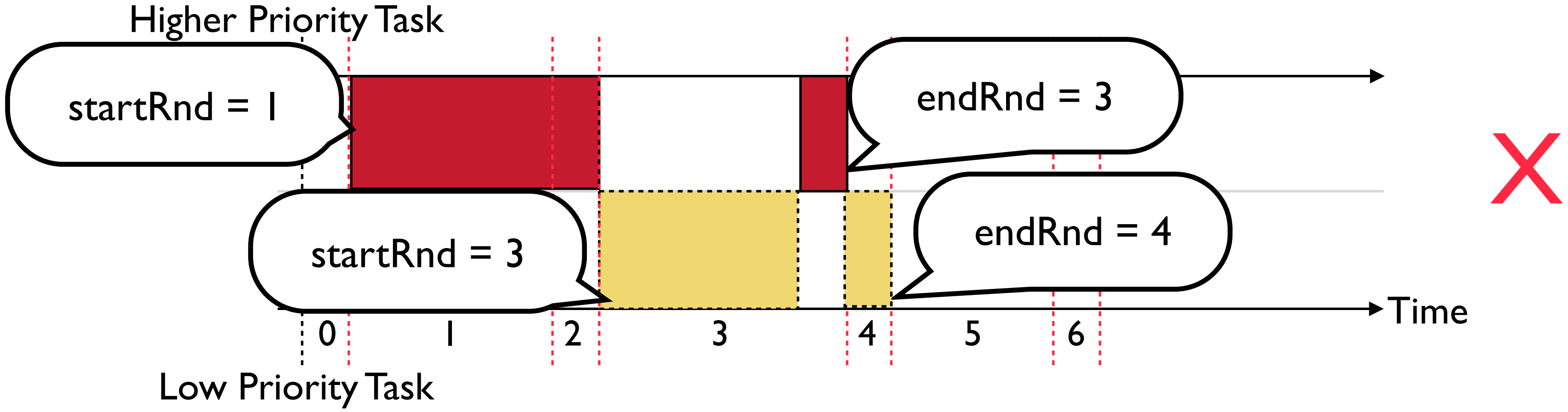


MonoSeq Sequentialization

Add constraints to enforce **legal** job scheduling

2. Jobs are well-nested:





MONOSEQ Sequentialization

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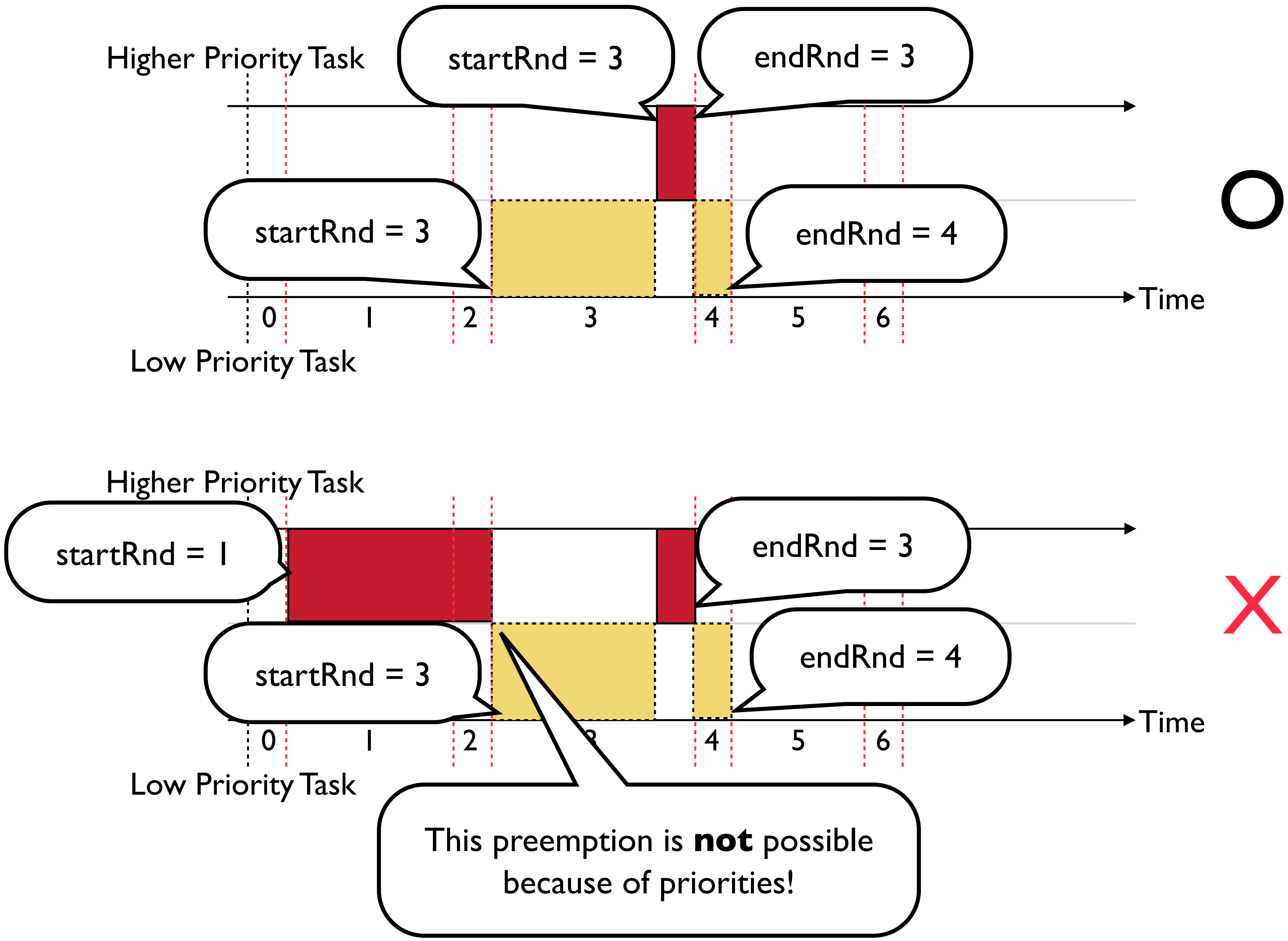

MonoSeq Sequentialization

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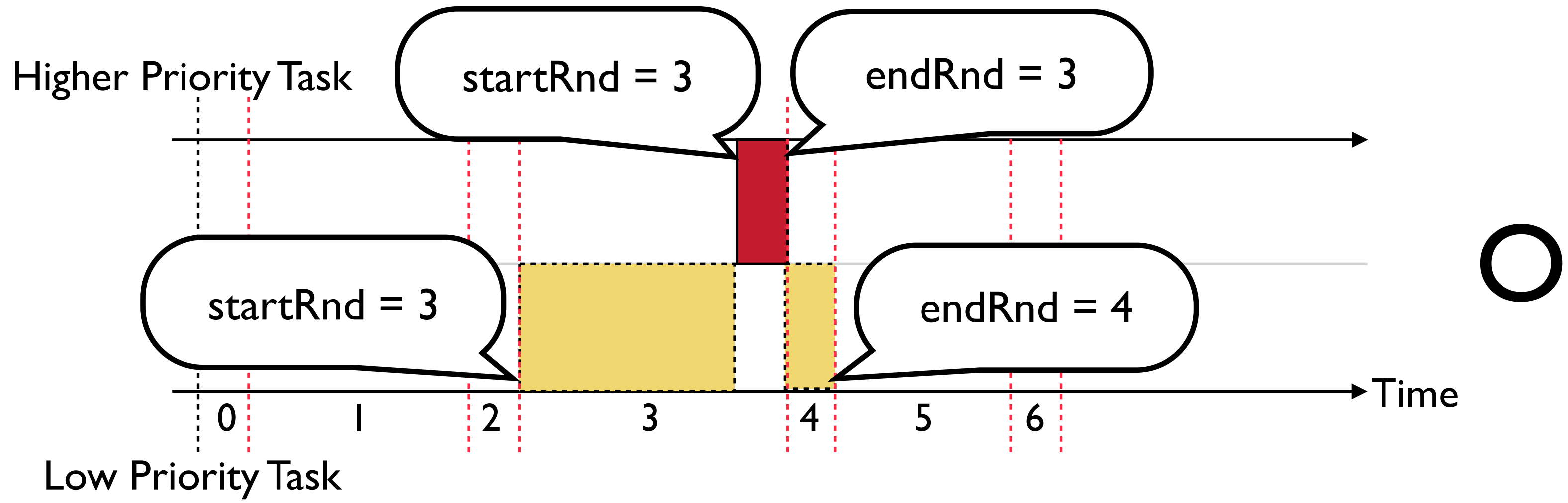

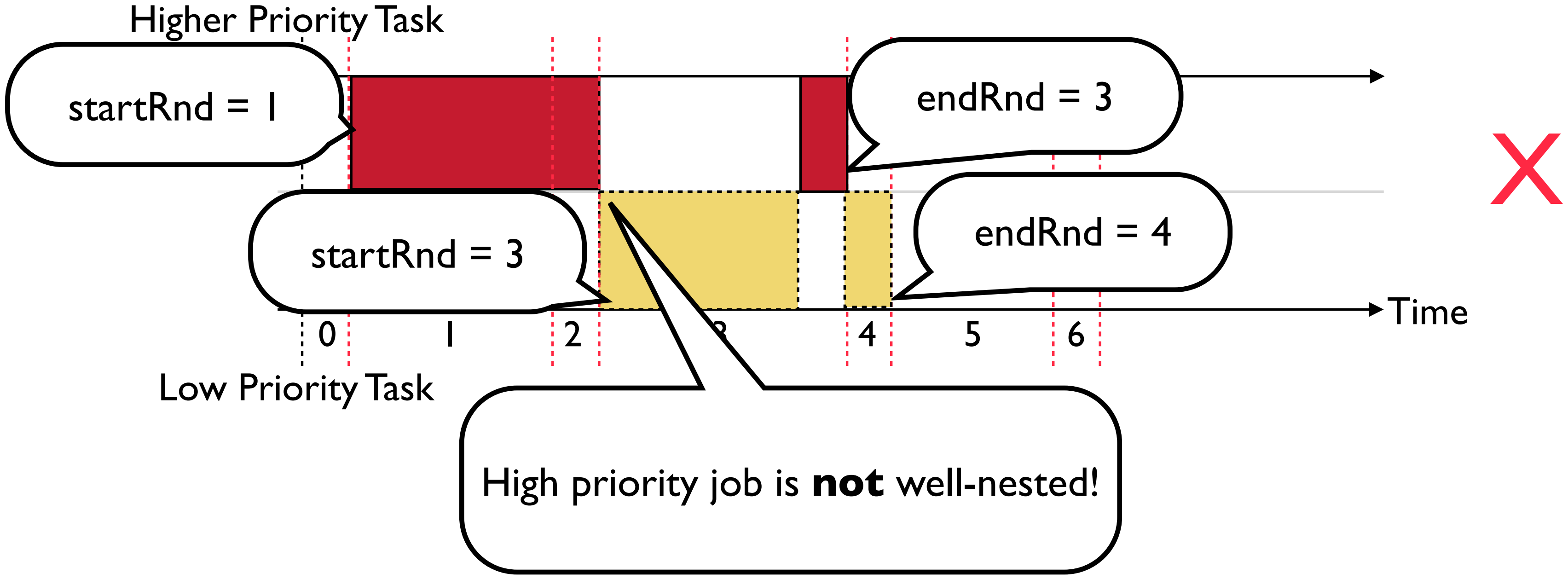

MonoSeq Sequentialization

Add constraints to enforce **legal** job scheduling

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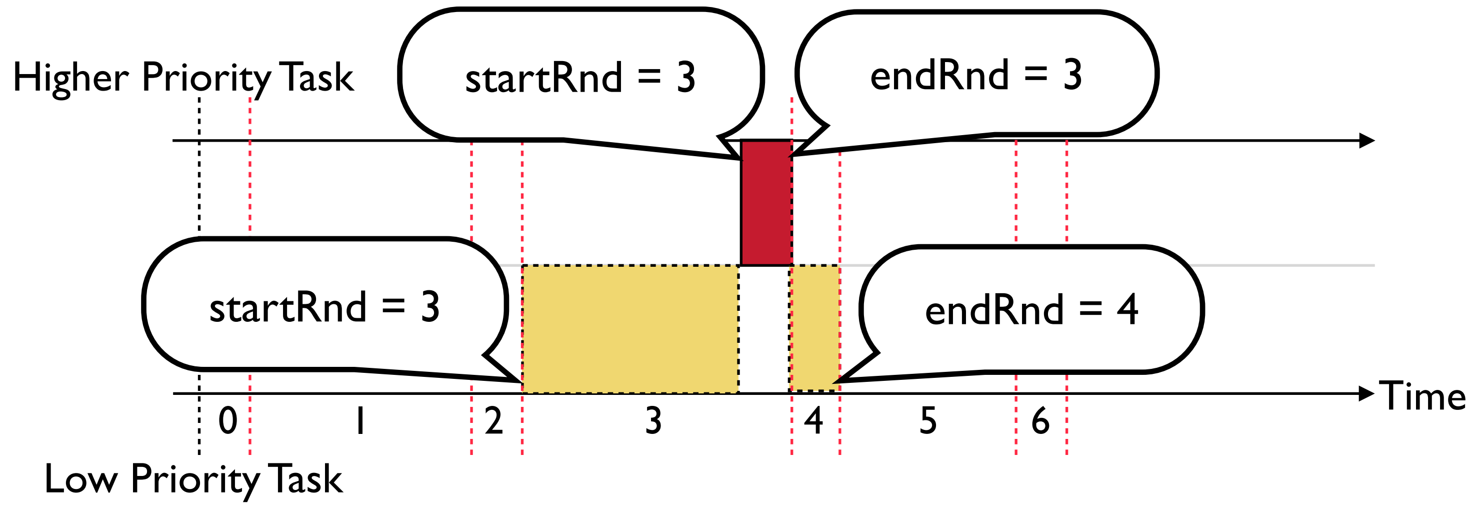

// Jobs are well-nested

$$\forall t_1 \in \mathsf{T}, t_2 \in \mathsf{T}, j_1 \in \mathsf{J}(t_1), j_2 \in \mathsf{J}(t_2) \cdot$$
$$\text{assume}($$
$$\big(t_1 < t_2 \wedge$$
$$start[t_1][j_1] \leq end[t_2][j_2] \wedge$$
$$start[t_2][j_2] \leq end[t_1][j_1]\big) \Rightarrow$$
$$(start[t_1][j_1] \leq start[t_2][j_2] \leq end[t_2][j_2] < end[t_1][j_1]))$$

MonoSeq Sequentialization

Add constraints to enforce **legal** job scheduling

3. Jobs respect preemption bounds:

$PB_{t_1}^{t_2}$ = Upper bound on the number of times t_1 can preempt t_2 .

$\textsc{MonoSeq}$ Sequentialization

Add constraints to enforce **legal** job scheduling

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RMA(Rate Monotonic Analysis) defines it

$PB_{t_1}^{t_2}$ = Upper bound on the number of times t_1 can preempt t_2 .

$\textsc{MonoSeq}$ Sequentialization

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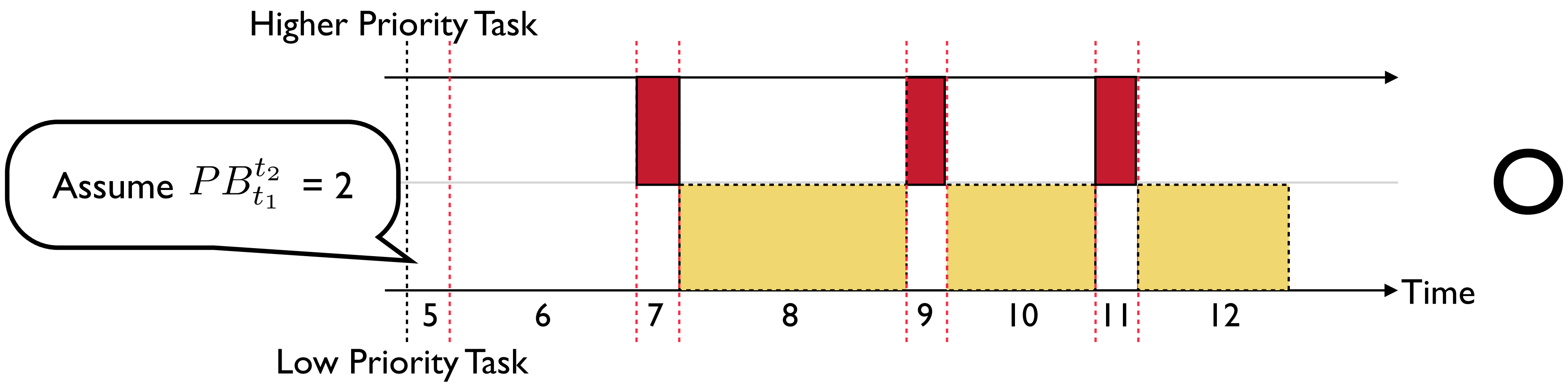

MonoSeq Sequentialization

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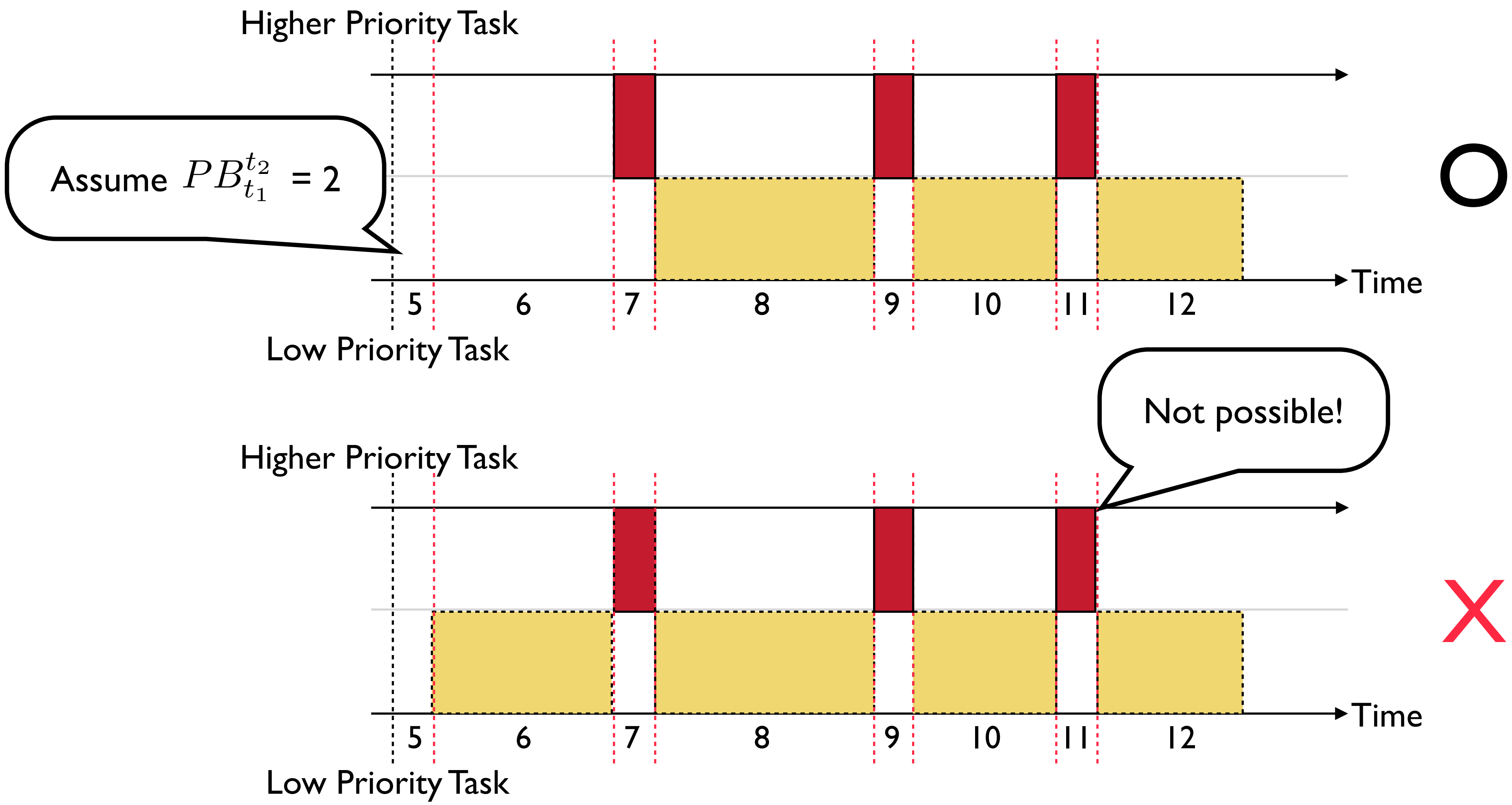

MonoSeq Sequentialization

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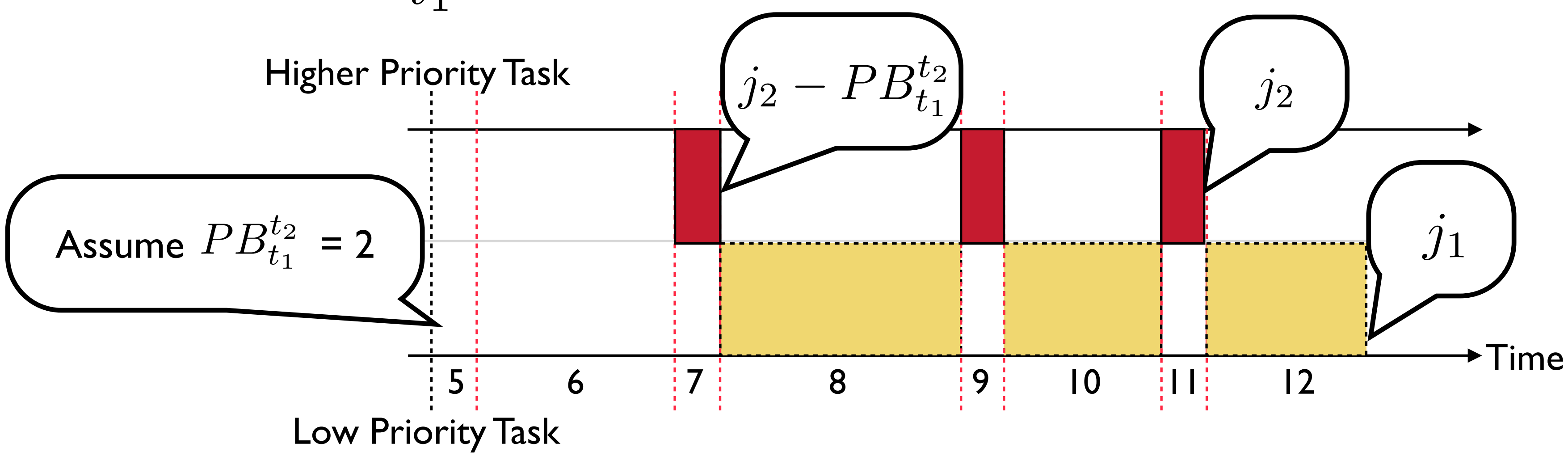

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$$\text{assume}($$
$$\big(t_1 < t_2 \wedge j_2 \geq PB_{t_1}^{t_2} \wedge$$
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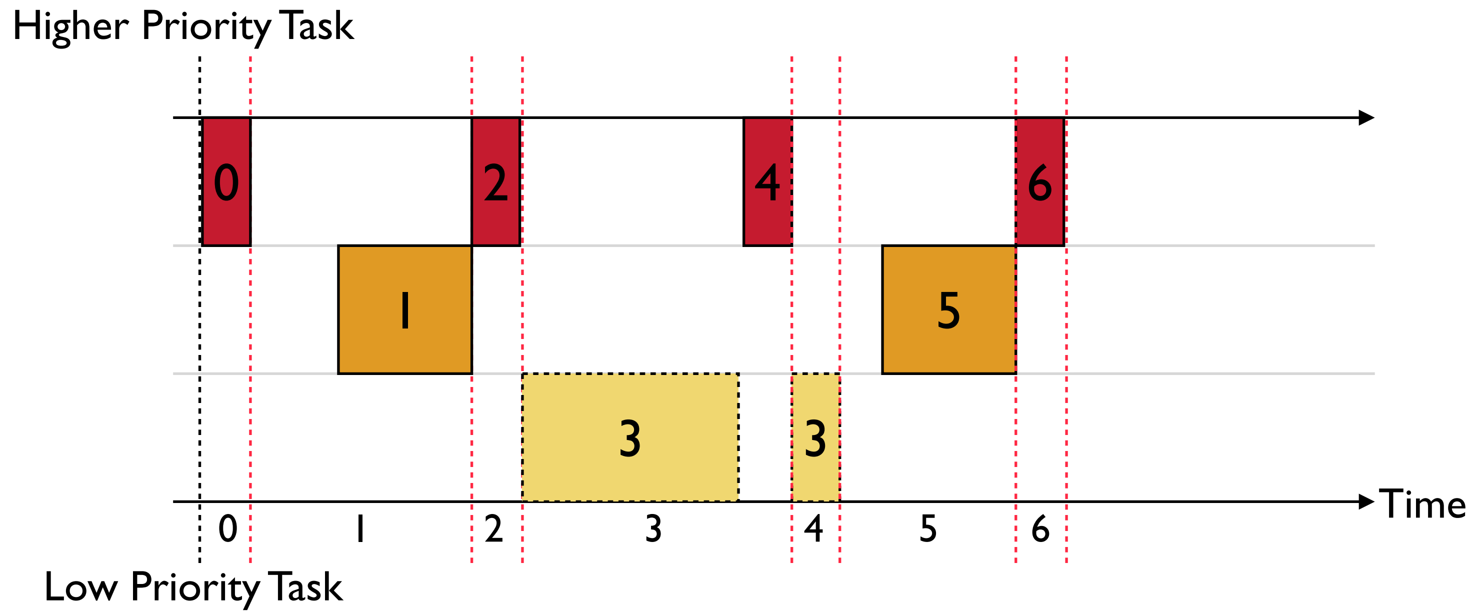
MonoSeq Sequentialization

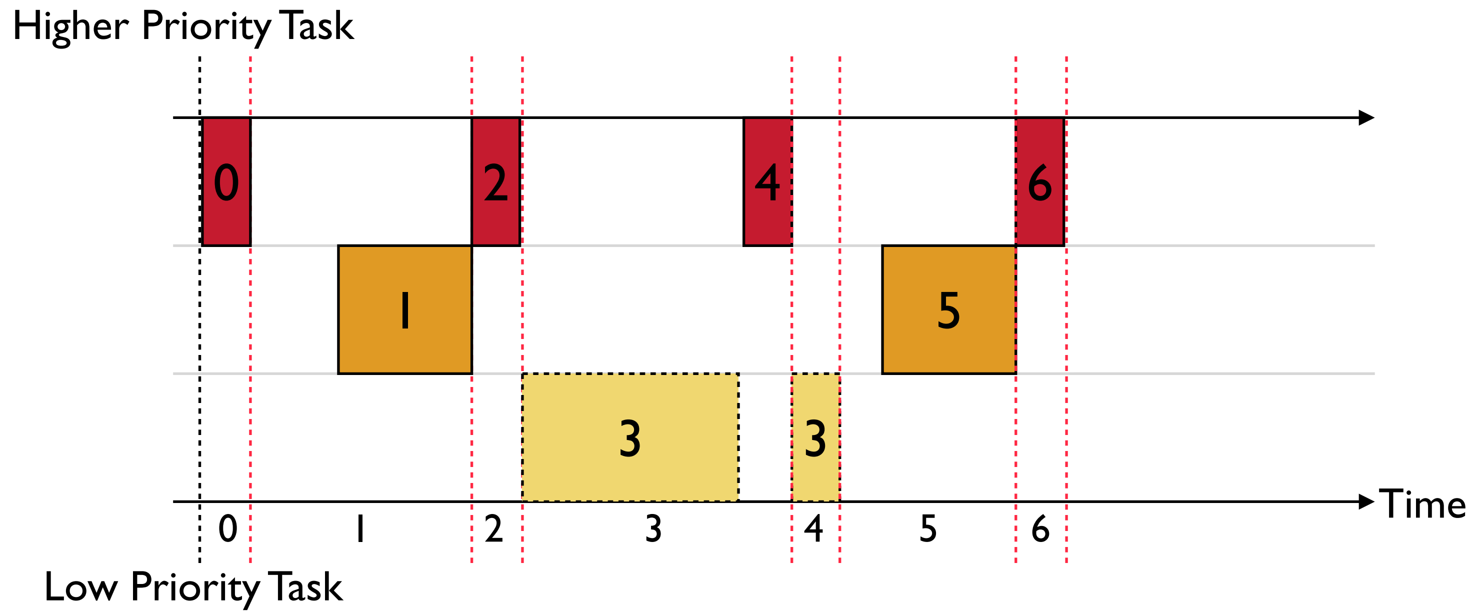
Sequential Execution



Replace accesses to global variable g with g[rnd]

MonoSeq Sequentialization

Sequential Execution

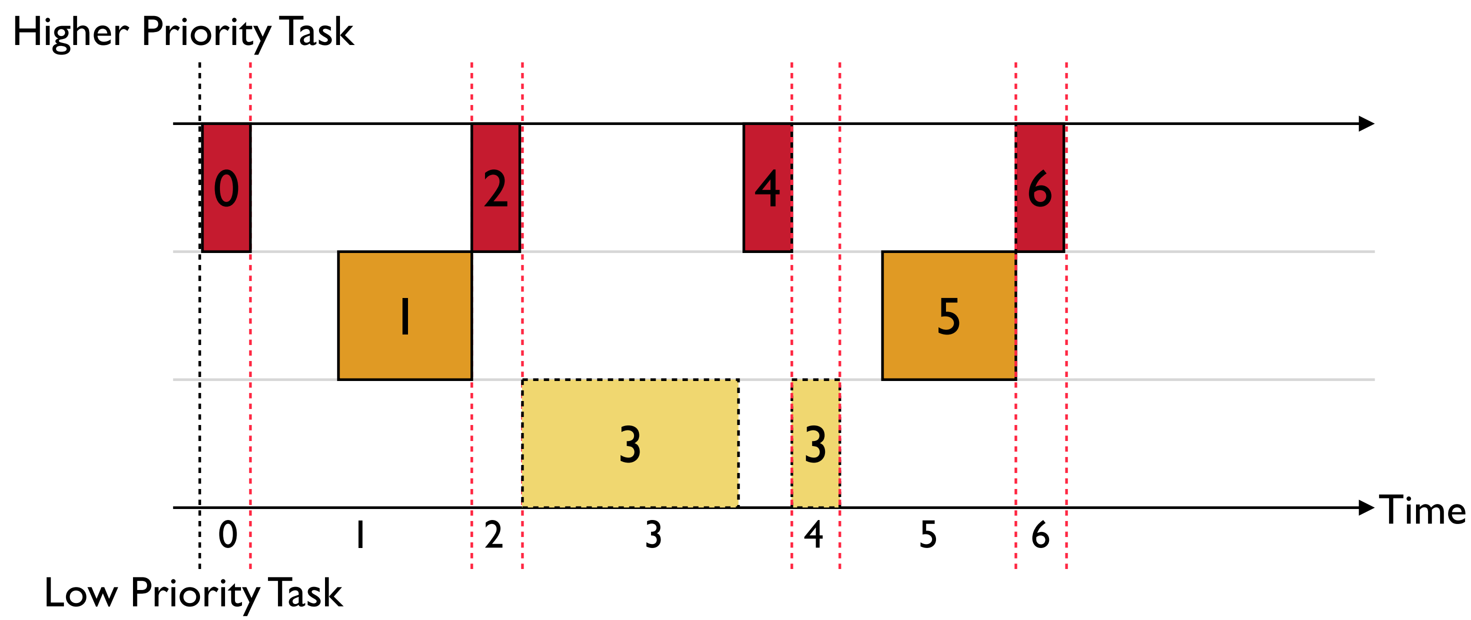


Replace accesses to global variable g with g[rnd]

```
x := y + g;
```

MonoSeq Sequentialization

Sequential Execution



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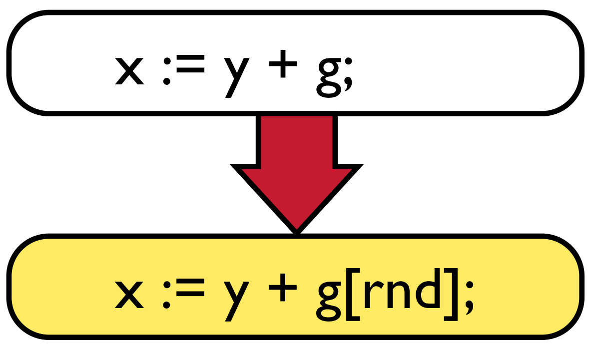
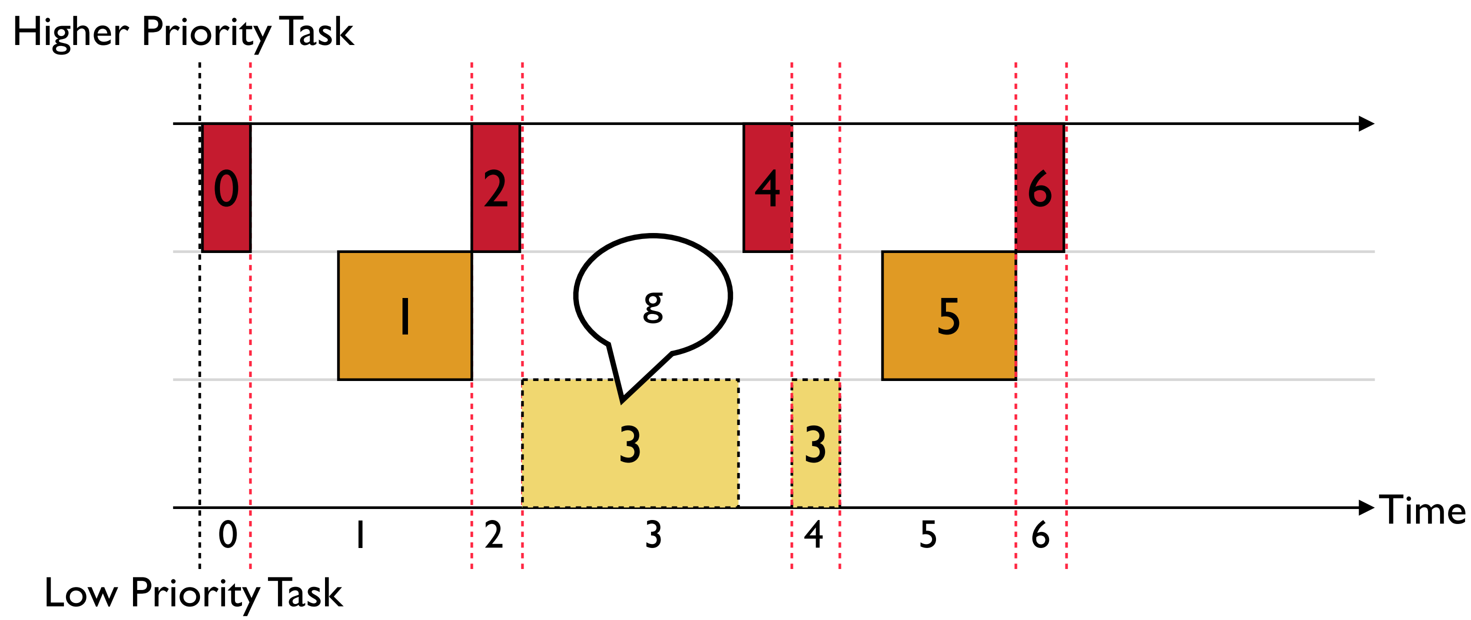


MONOSEQ Sequentialization

Sequential Execution



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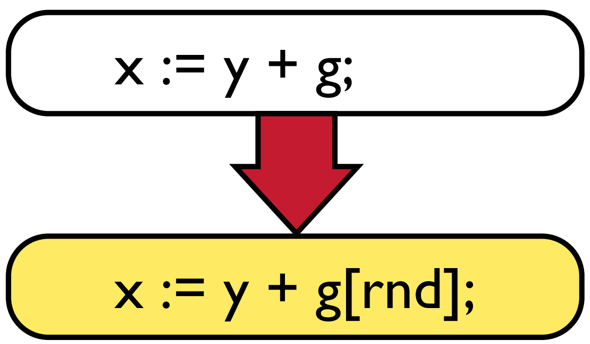
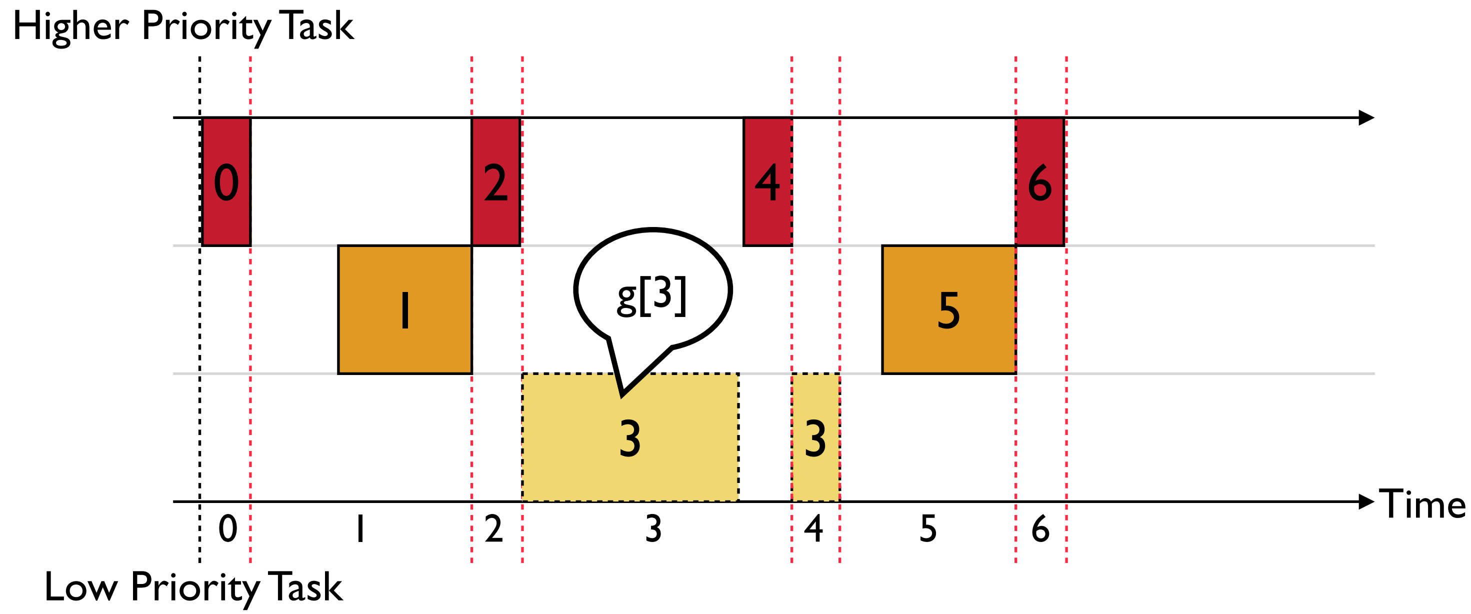


MONOSEQ Sequentialization

Sequential Execution



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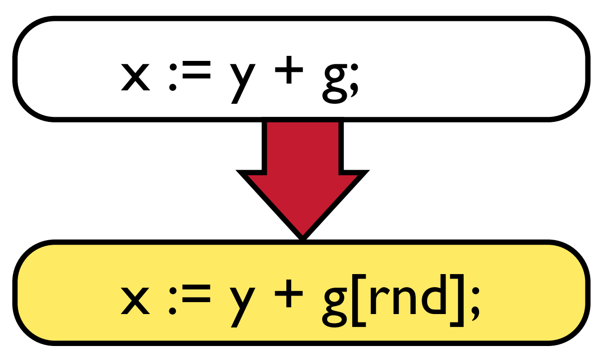
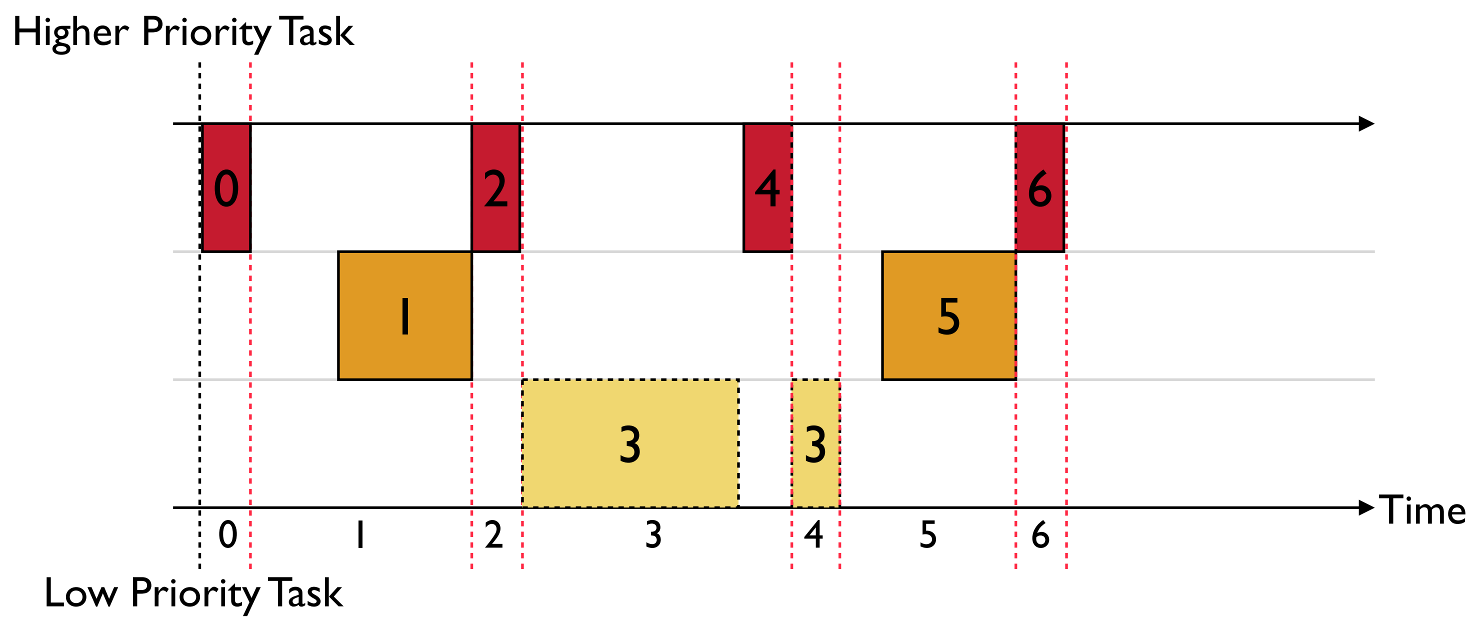
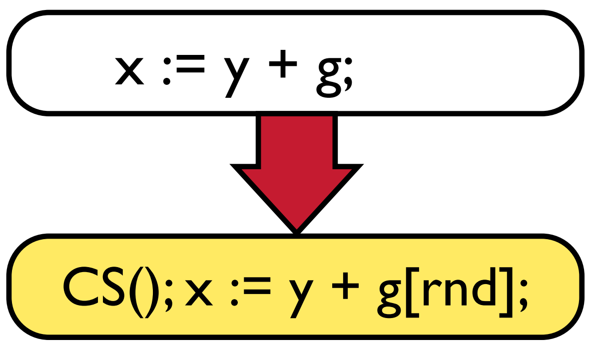


MONOSEQ Sequentialization

Sequential Execution

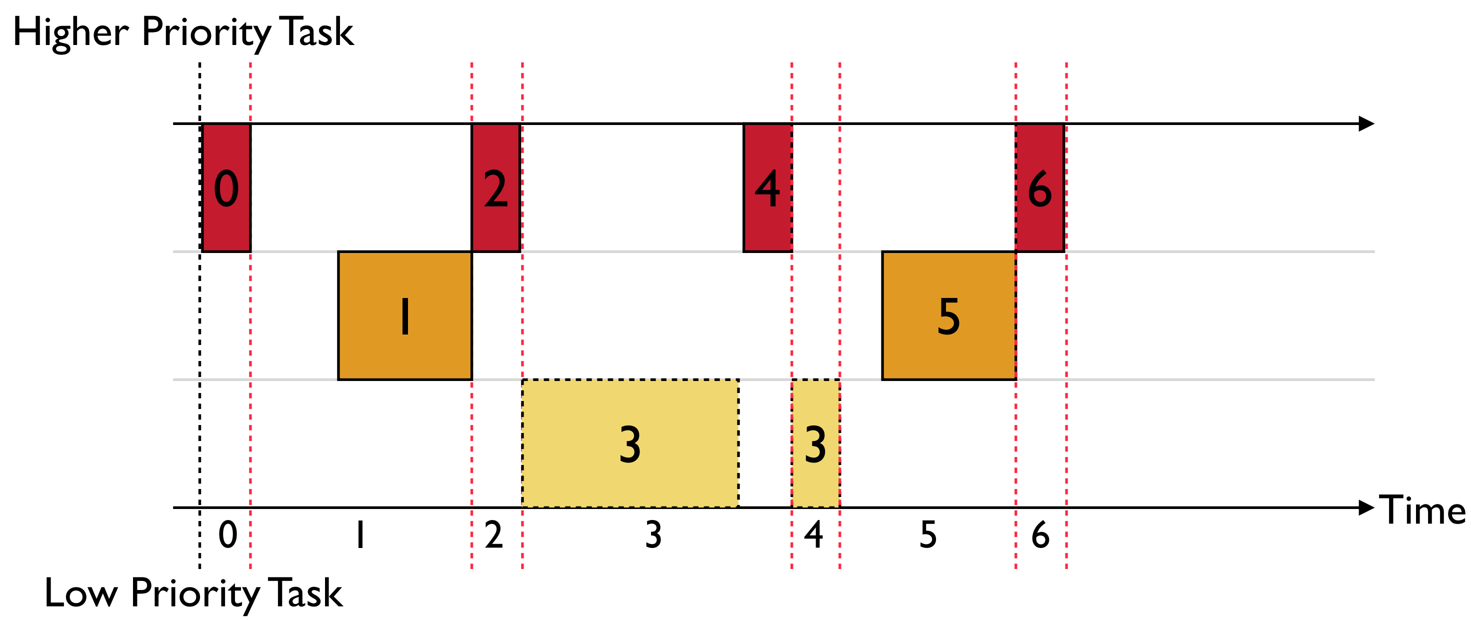


Add non-deterministic **context-switching** to statements



MonoSeq Sequentialization

Sequential Execution



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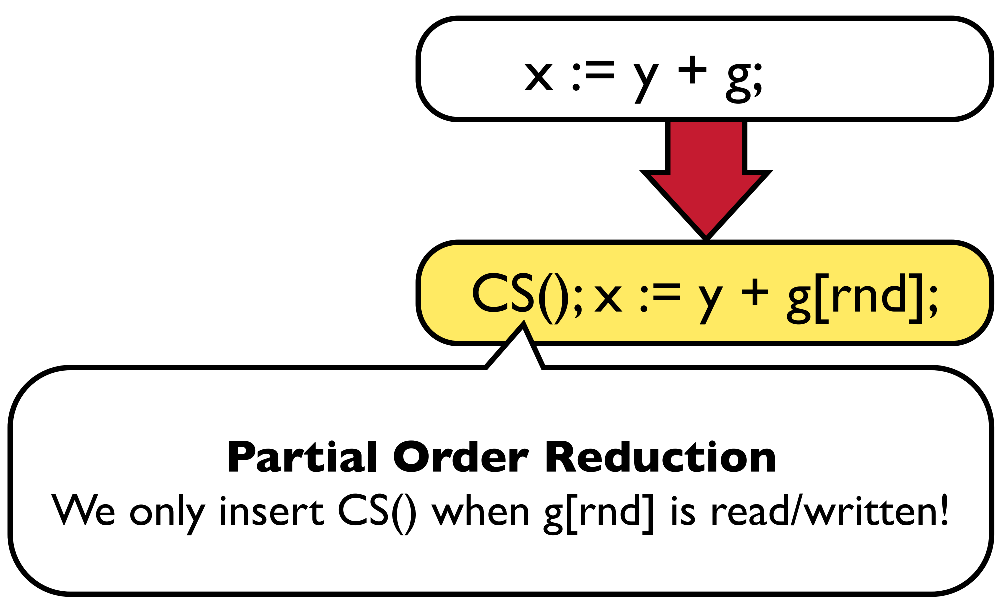
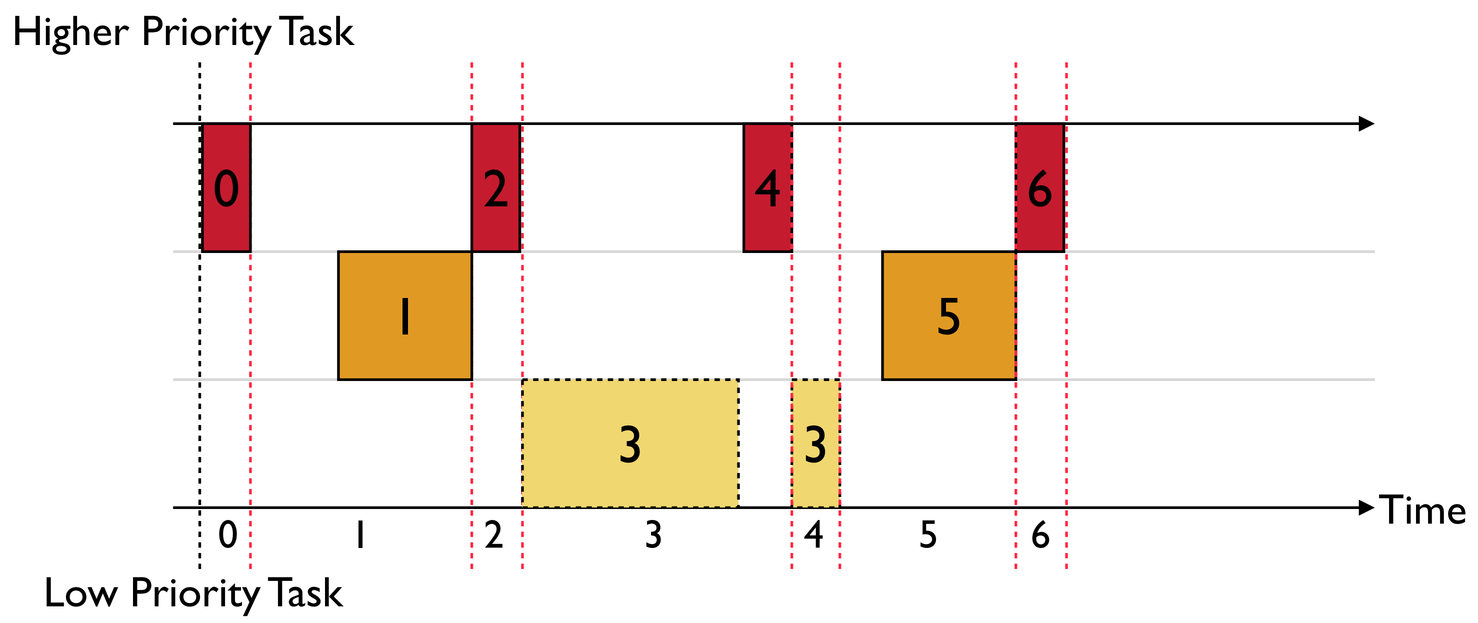


MONOSEQ Sequentialization

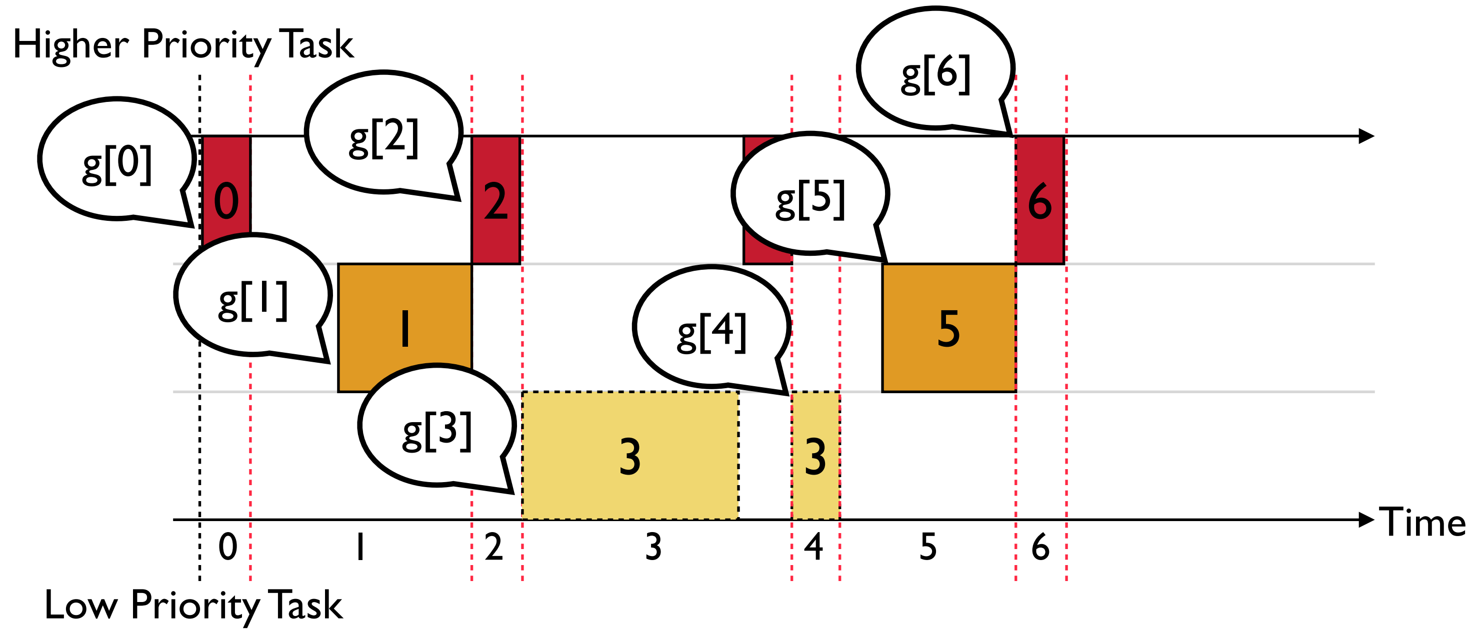
Sequential Execution



Guess non-deterministic initial value of each global in each round.

MonoSeq Sequentialization

Sequential Execution



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MonoSeq Sequentialization

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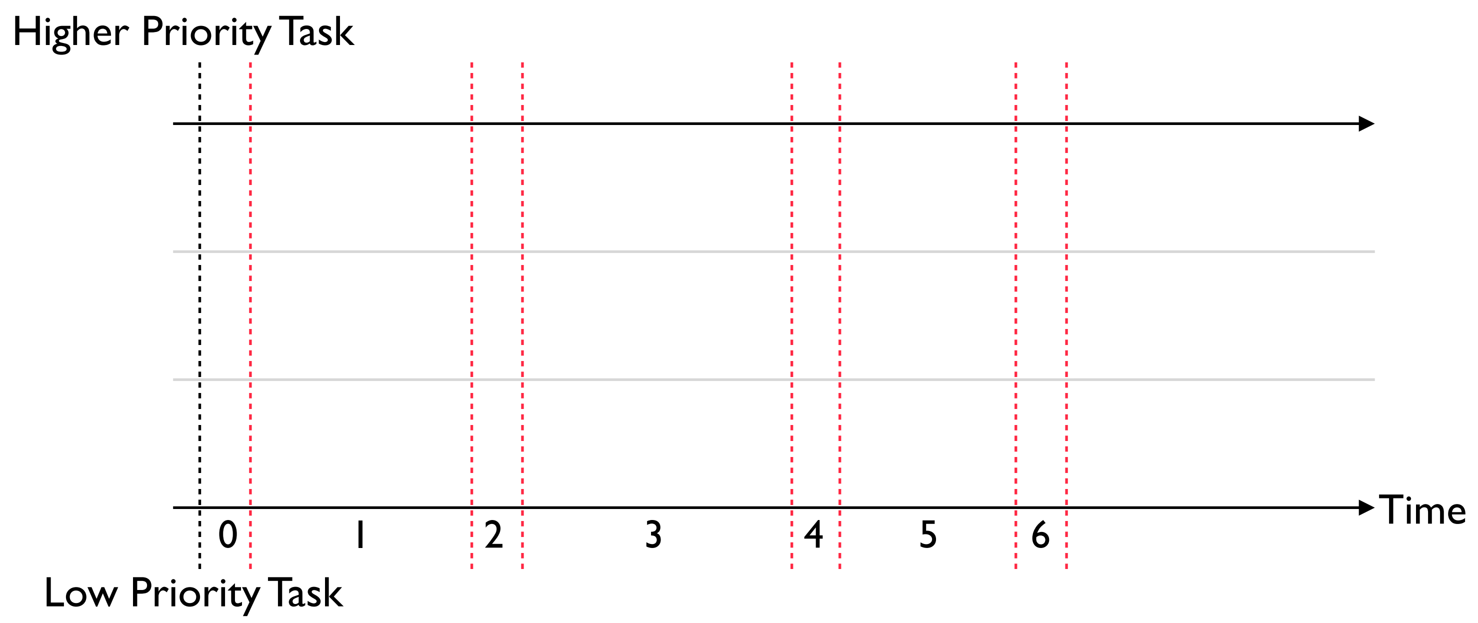
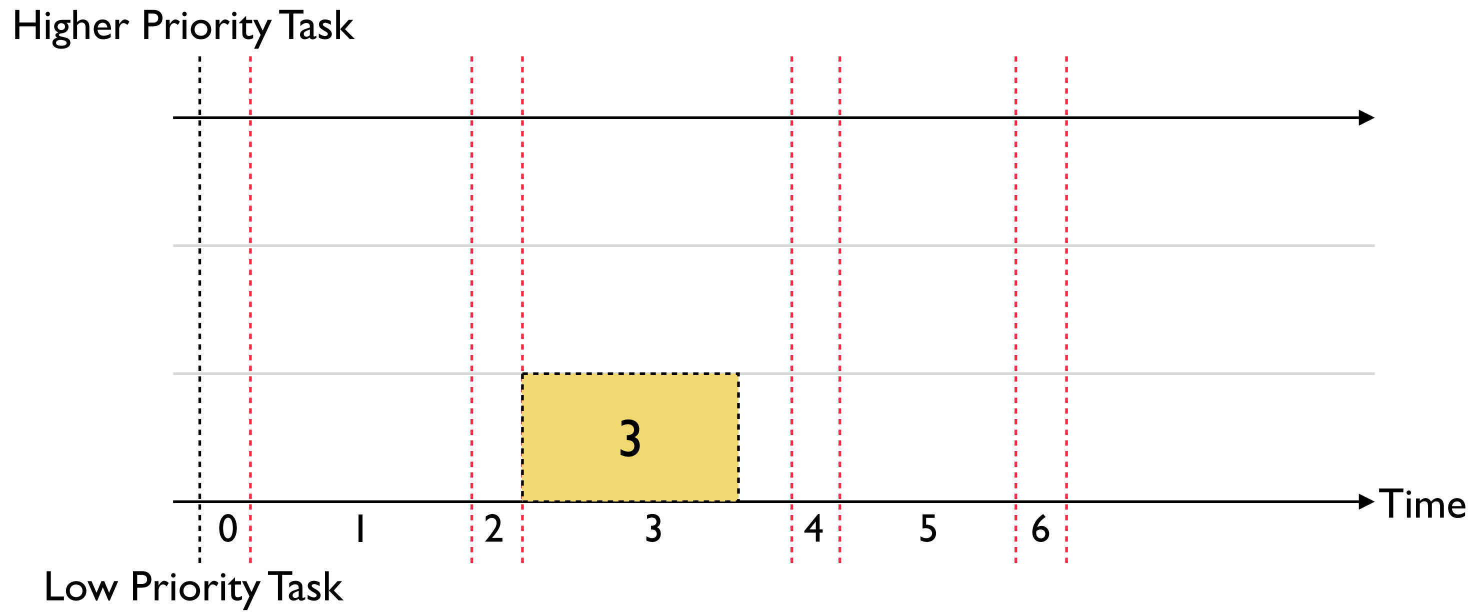


Guess non-deterministic initial value of each global in each round.

Execute Task Body, from lower priority task to higher priority task

MonoSeq Sequentialization

Sequential Execution



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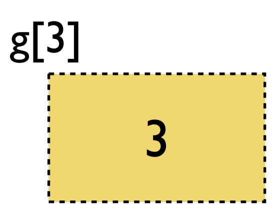
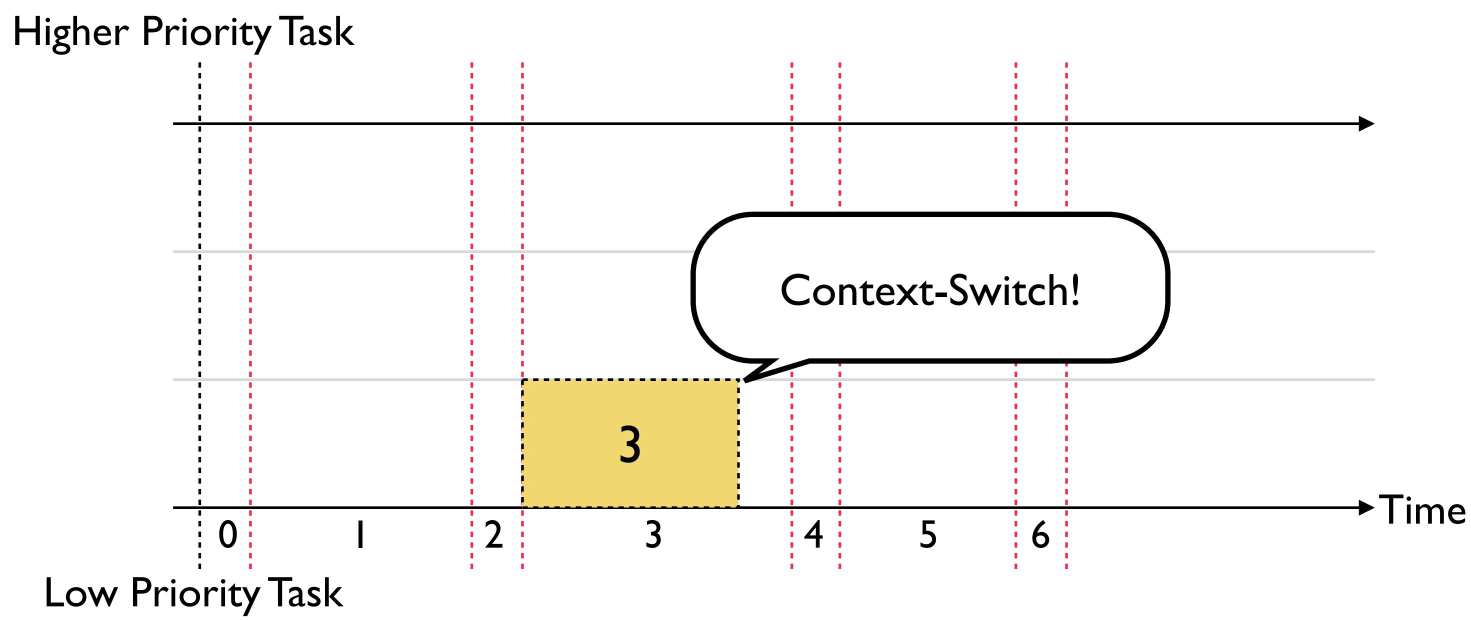


MonoSeq Sequentialization

Sequential Execution



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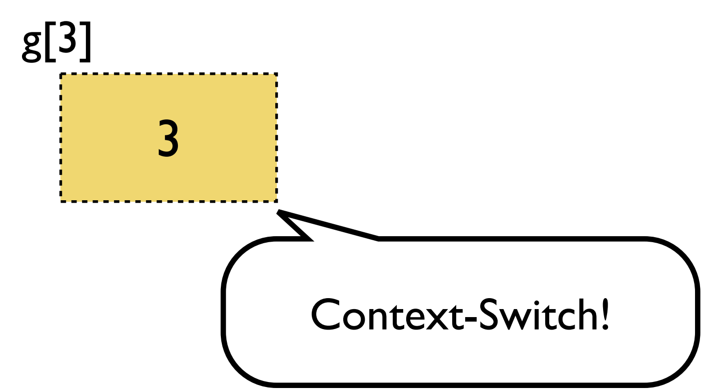
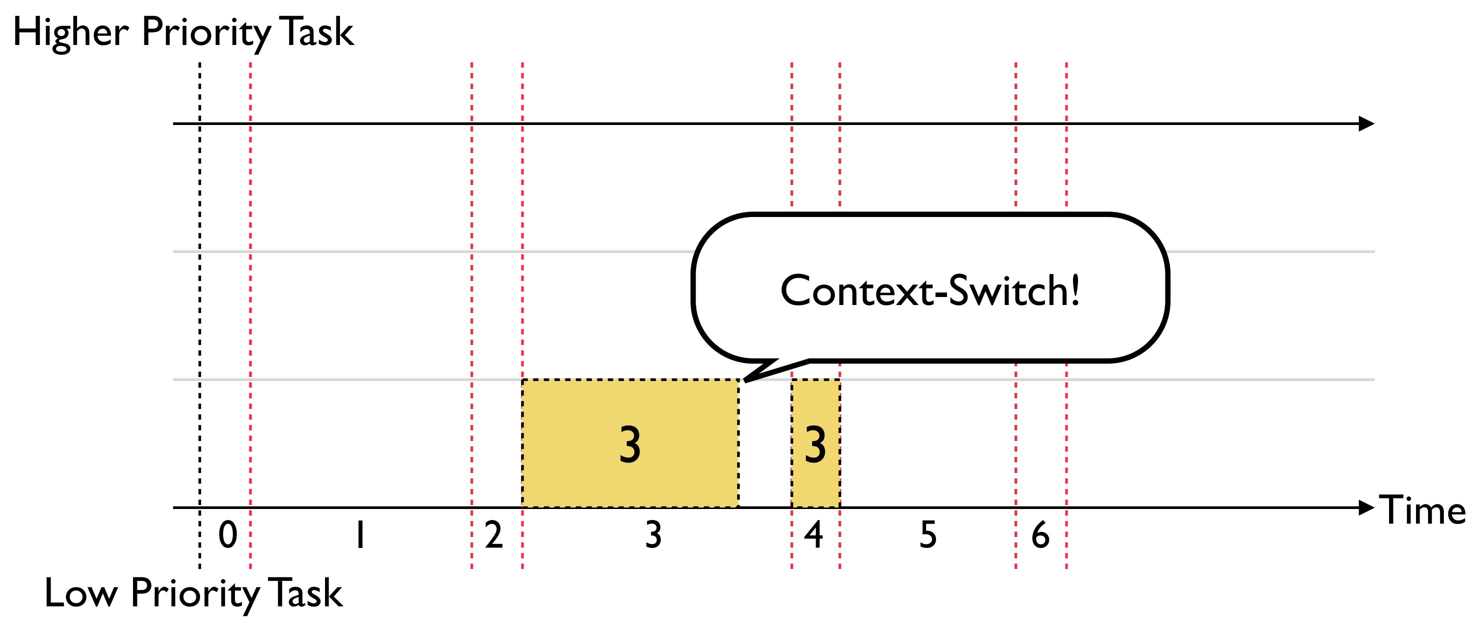
Execute Task Body, from lower priority task to higher priority task



MonoSeq Sequentialization

Sequential Execution



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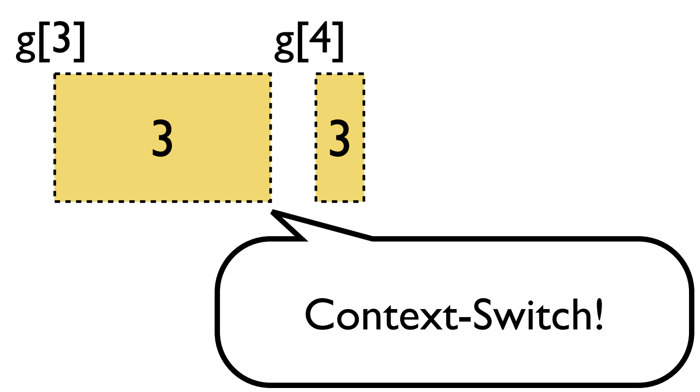
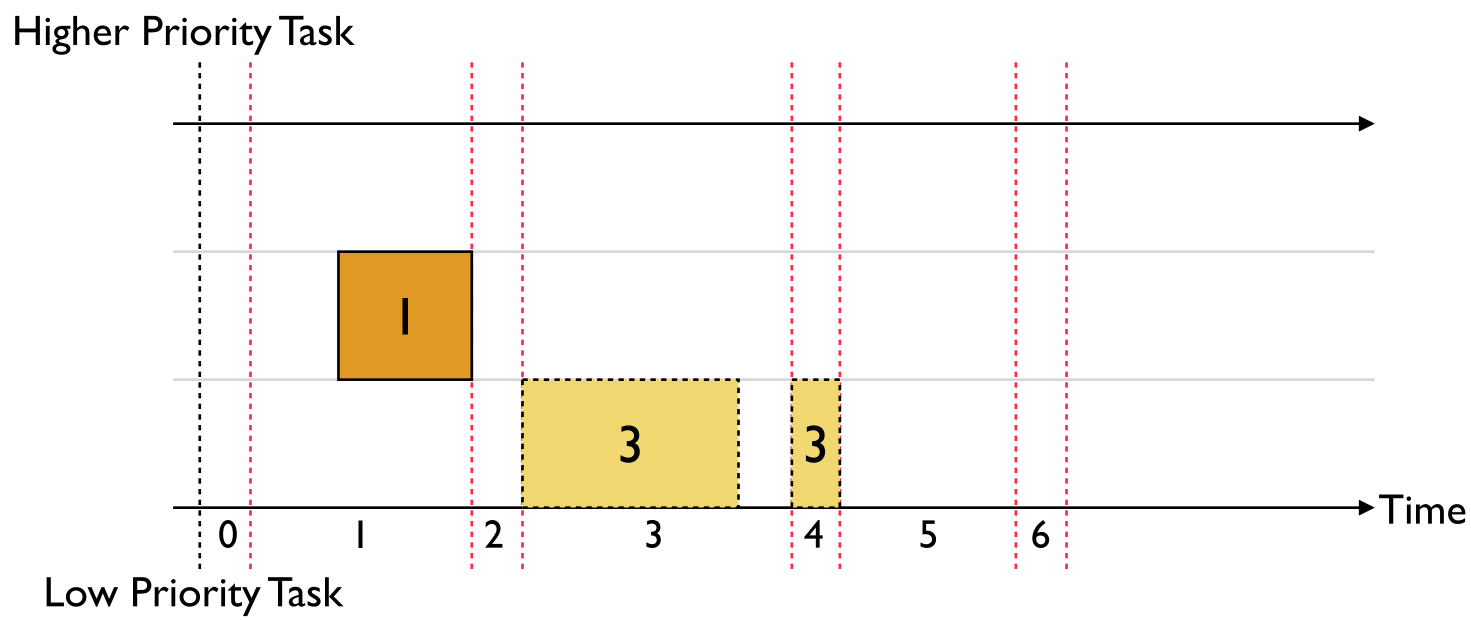


MonoSeq Sequentialization

Sequential Execution



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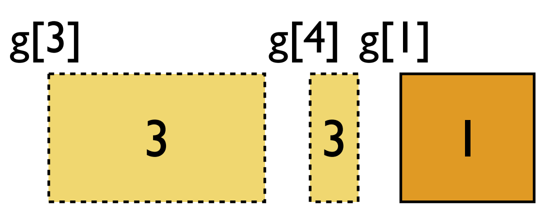
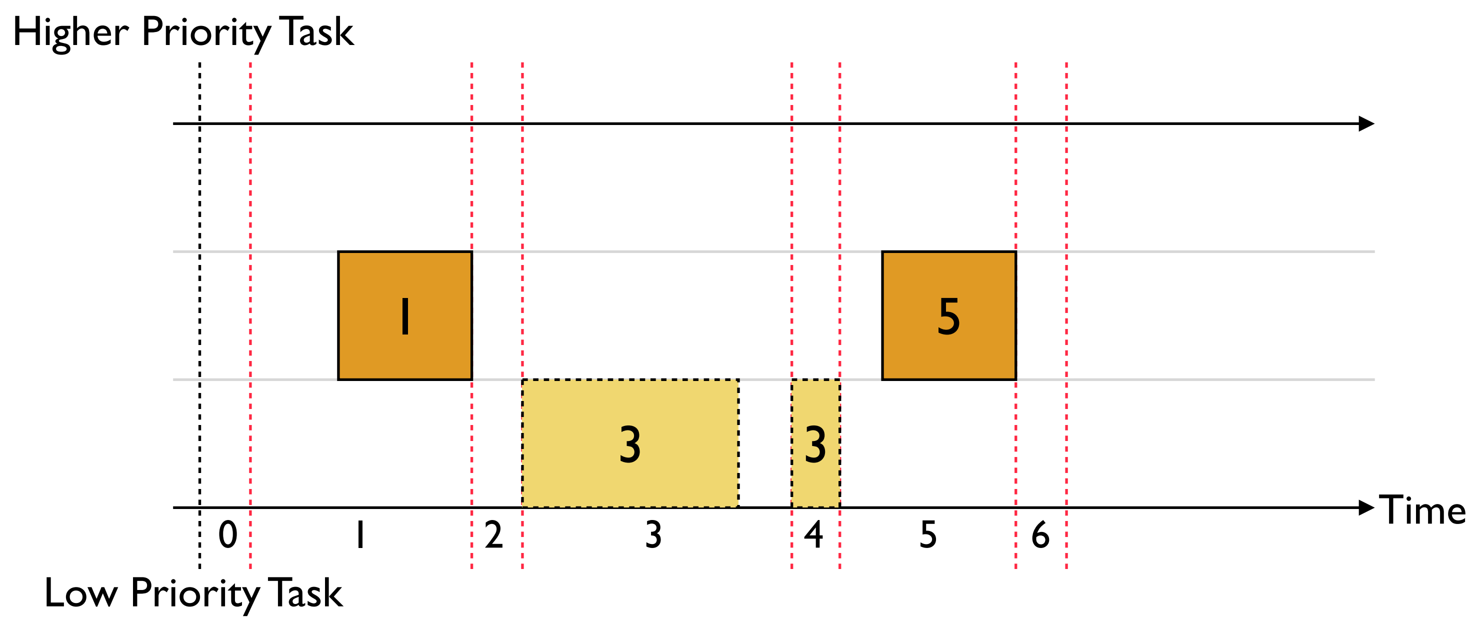


MonoSeq Sequentialization

Sequential Execution



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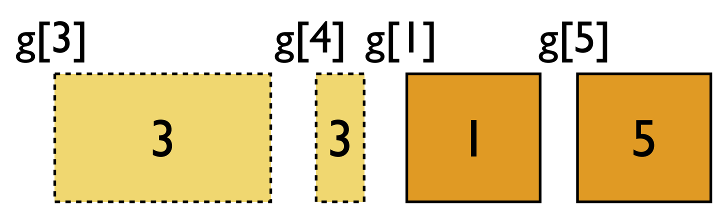
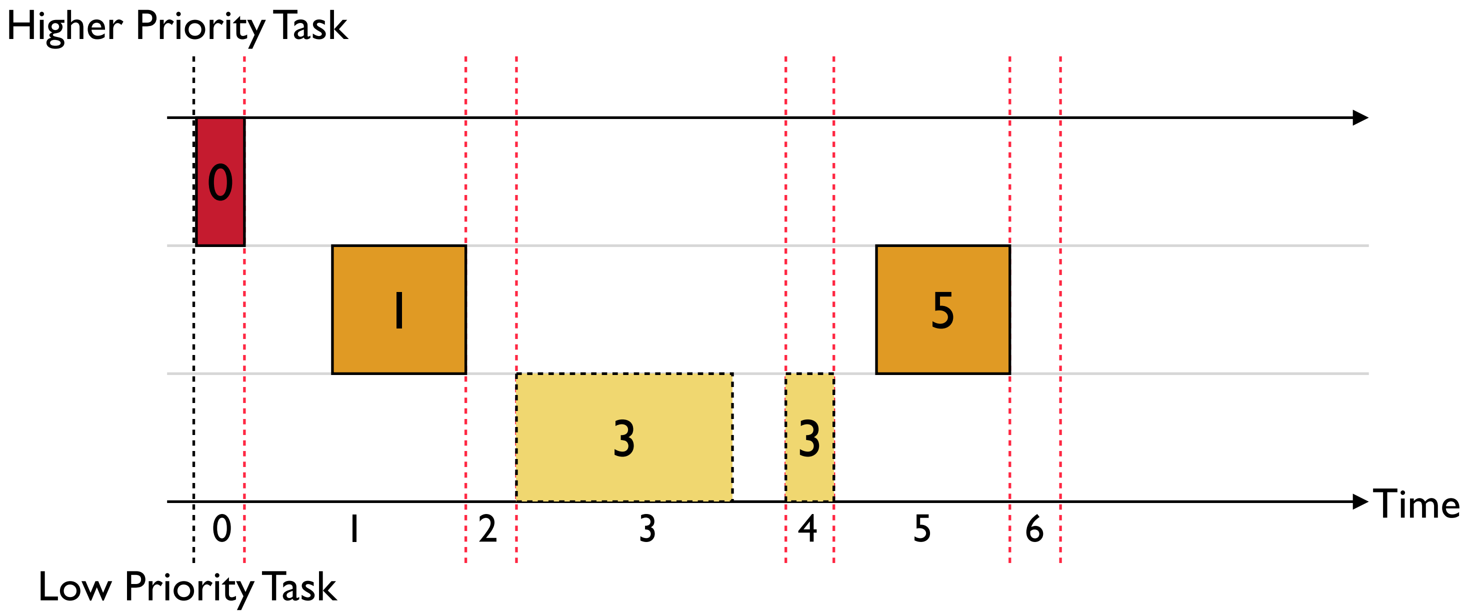


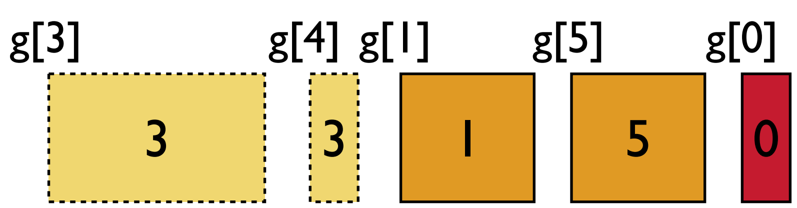
MONOSEQ Sequentialization

Sequential Execution



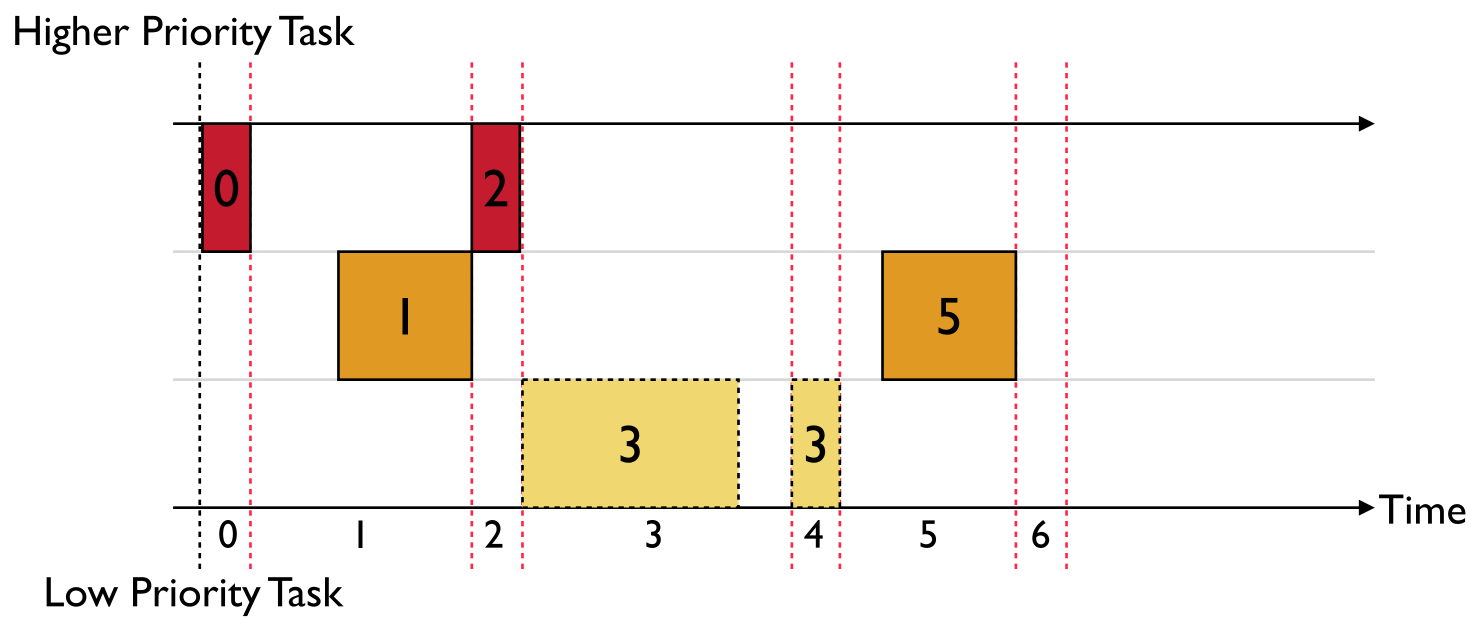
Guess non-deterministic initial value of each global in each round.

Execute Task Body, from lower priority task to higher priority task



MONOSEQ Sequentialization

Sequential Execution



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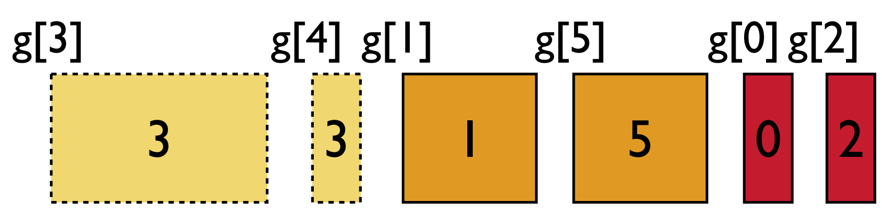
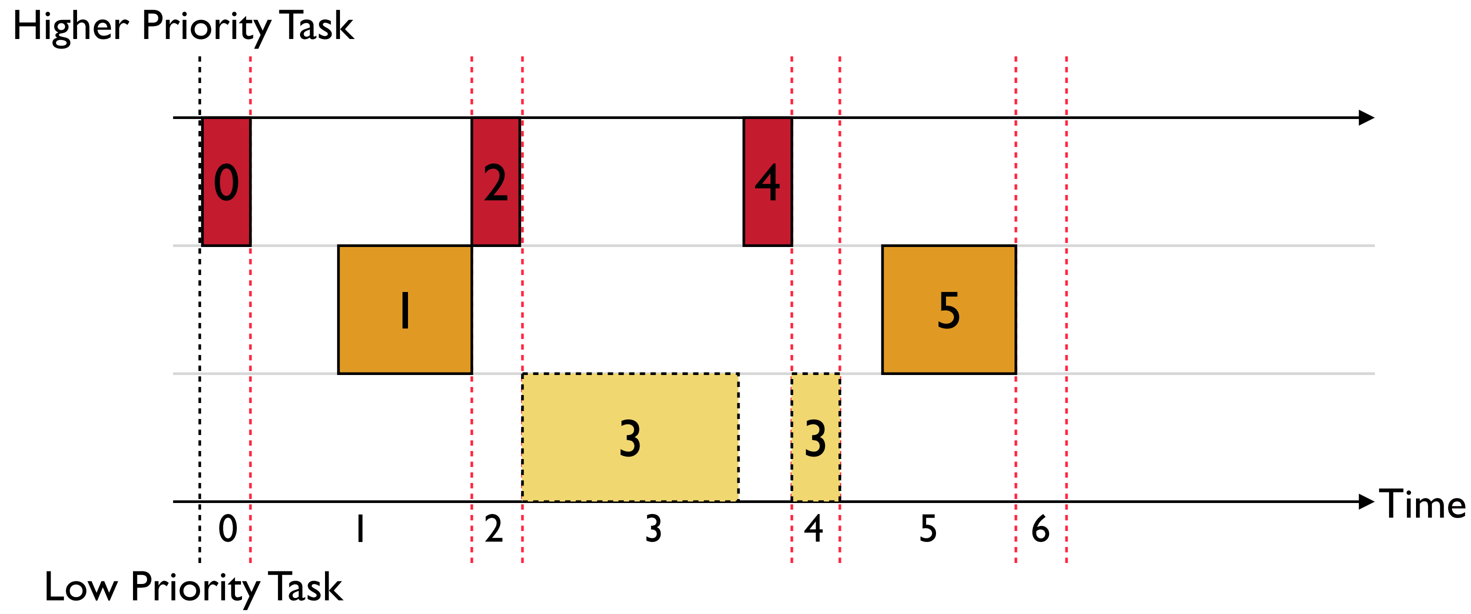


MONOSEQ Sequentialization

Sequential Execution



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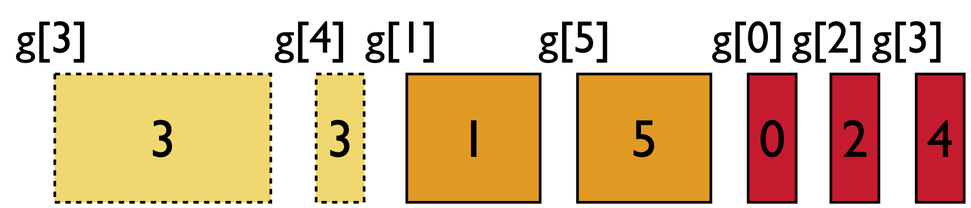
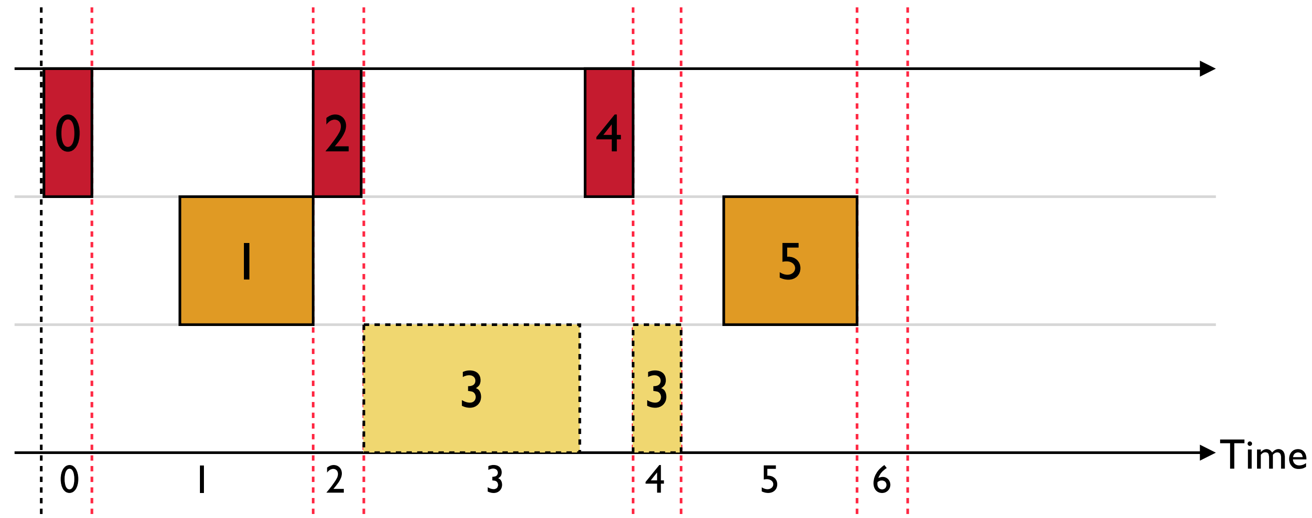


MonoSeq Sequentialization

Sequential Execution

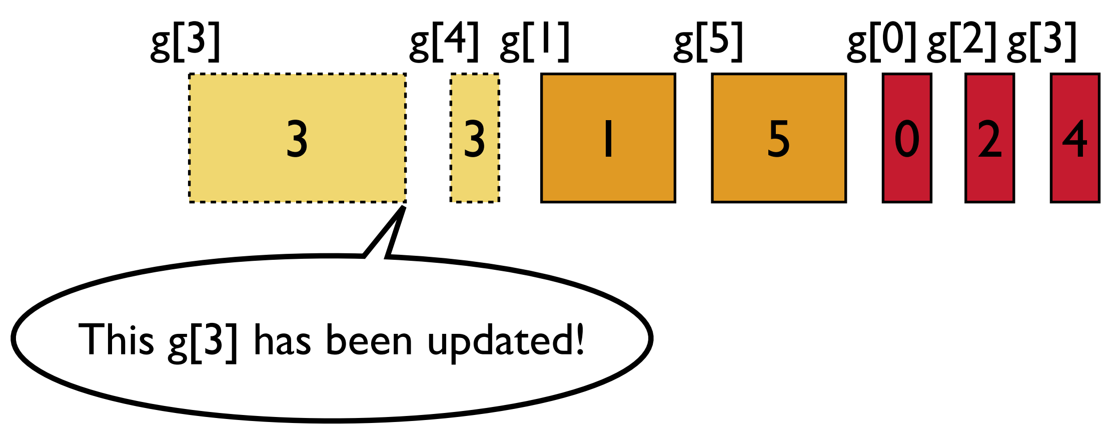
Higher Priority Task



Low Priority Task

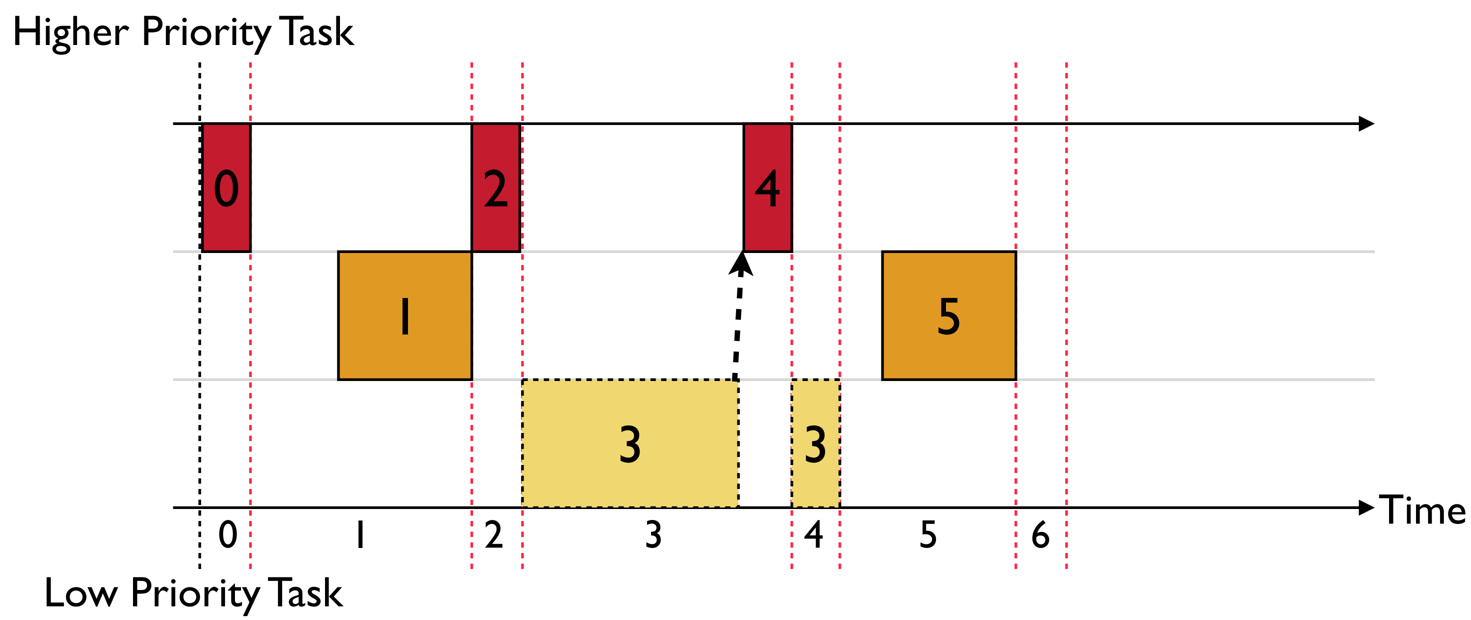
Guess non-deterministic initial value of each global in each round.

Execute Task Body, from lower priority task to higher priority task



MONOSEQ Sequentialization

Sequential Execution



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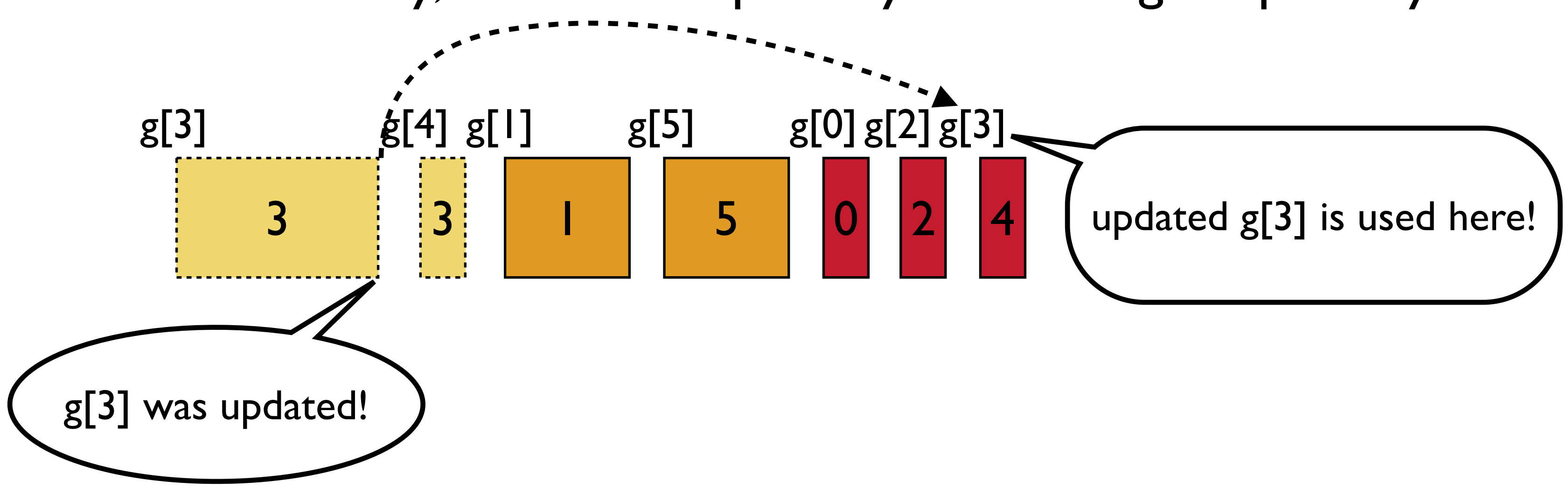
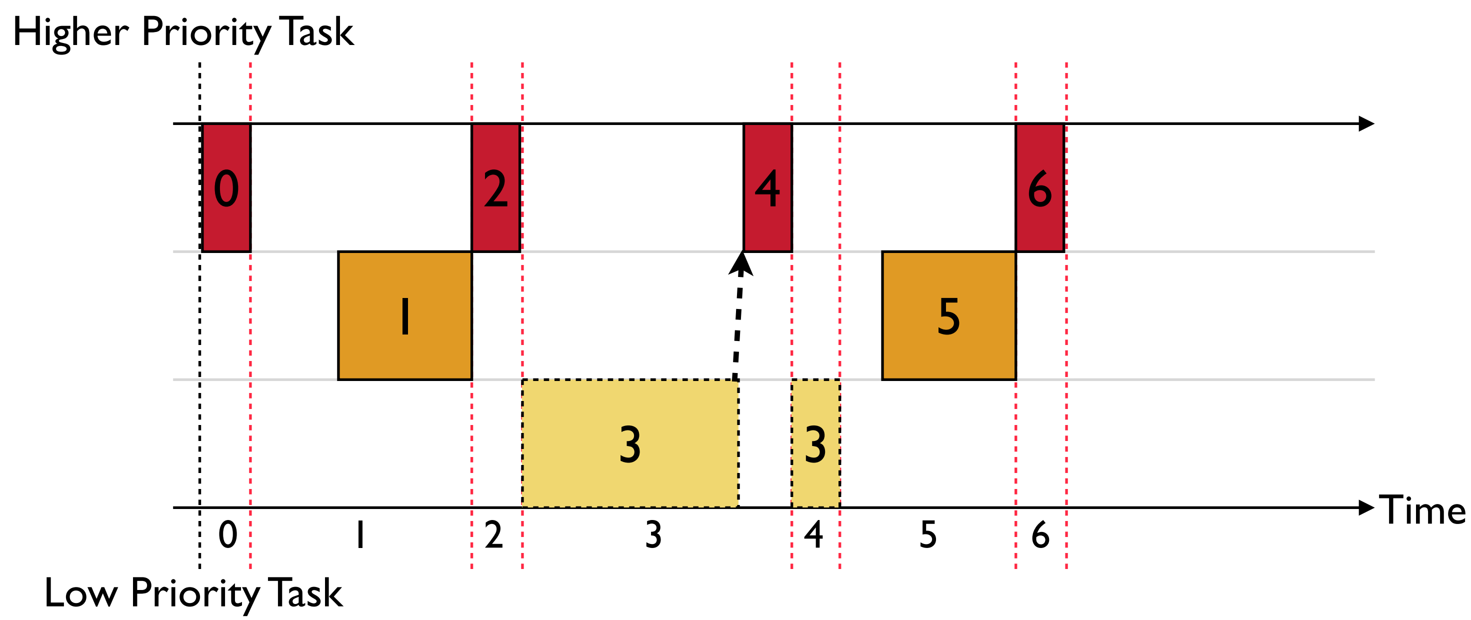


MonoSeq Sequentialization

Sequential Execution



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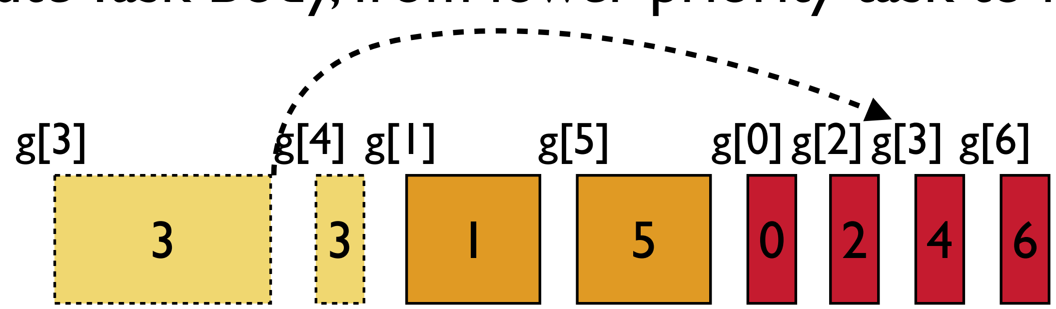
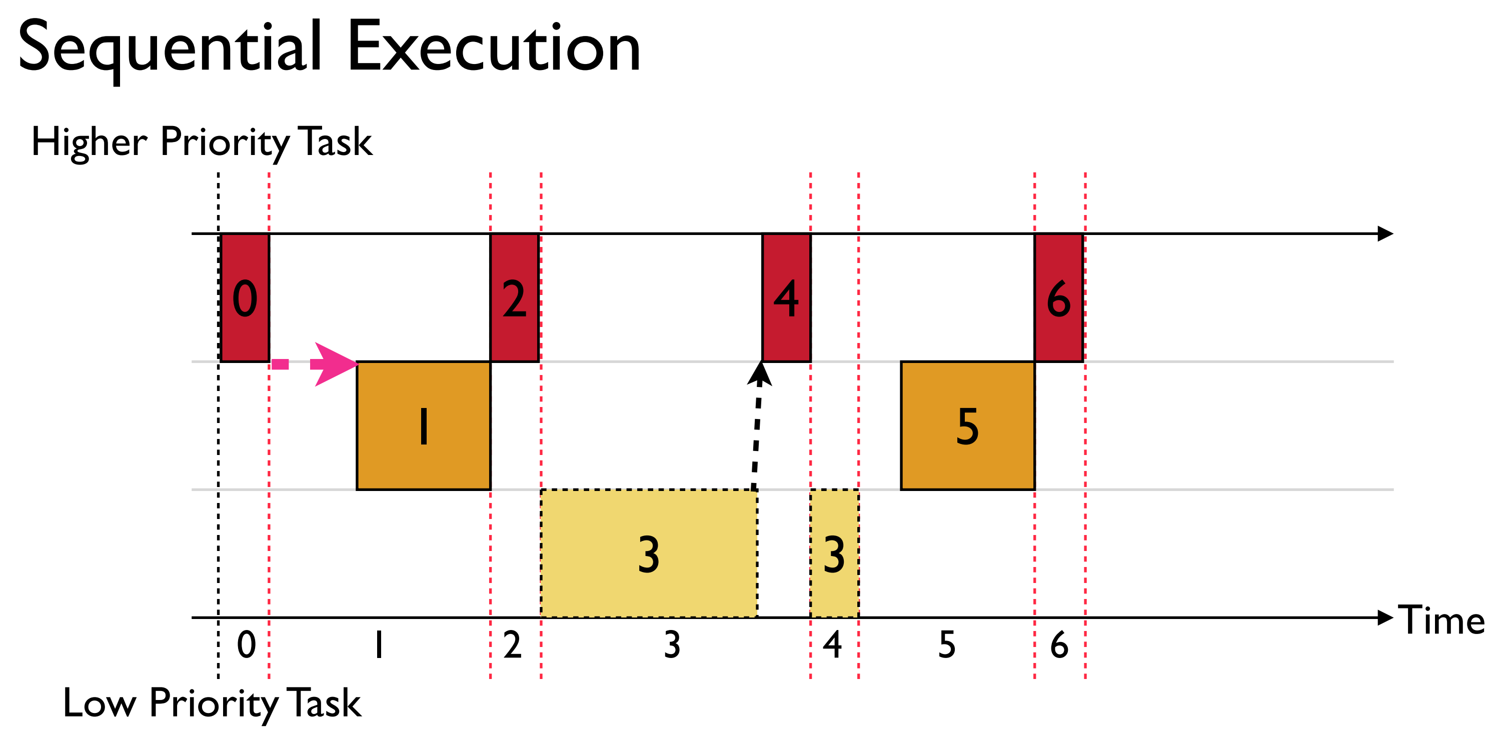


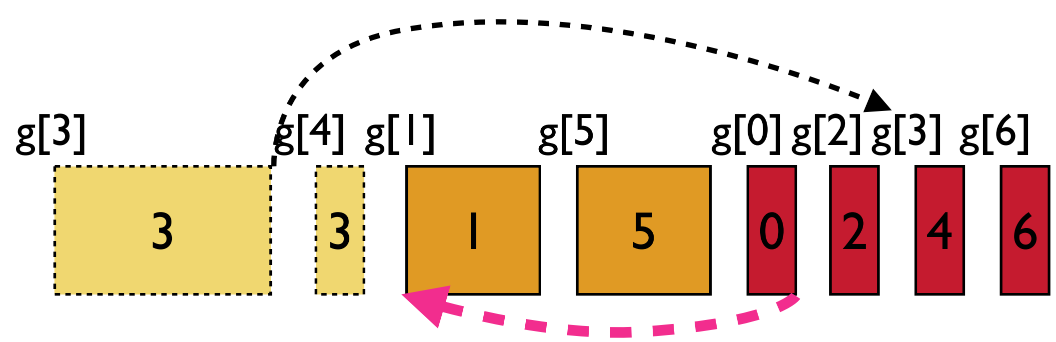
MONOSEQ Sequentialization



Guess non-deterministic initial value of each global in each round.

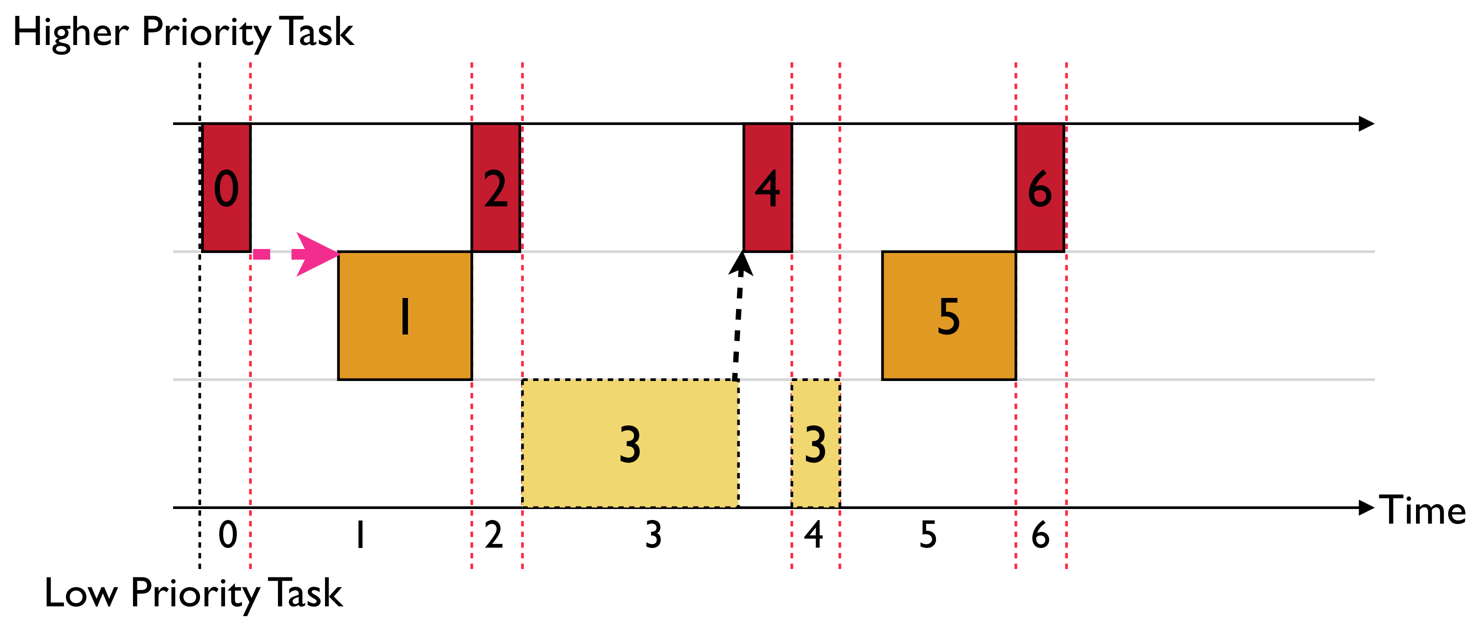
Execute Task Body, from lower priority task to higher priority task

Constraint the value of global variables g[ ]s to respect the order of execution



MonoSeq Sequentialization

Sequential Execution



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MONOSEQ Sequentialization

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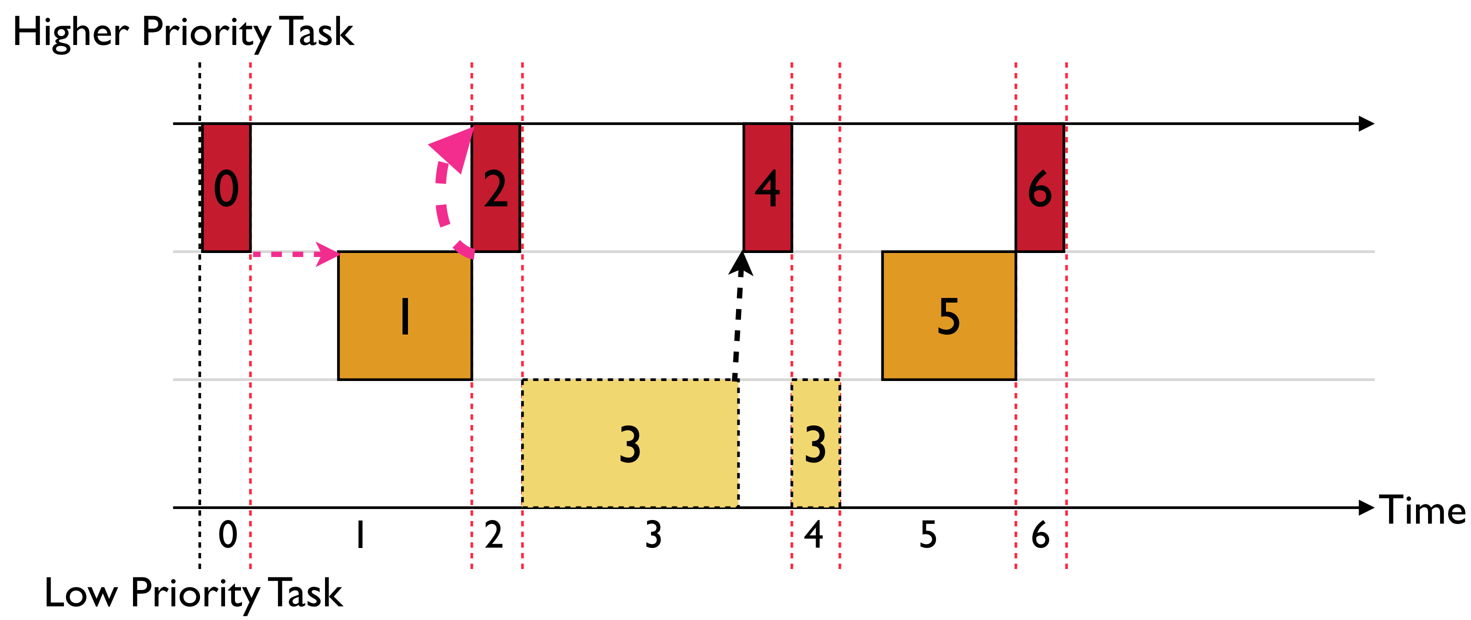

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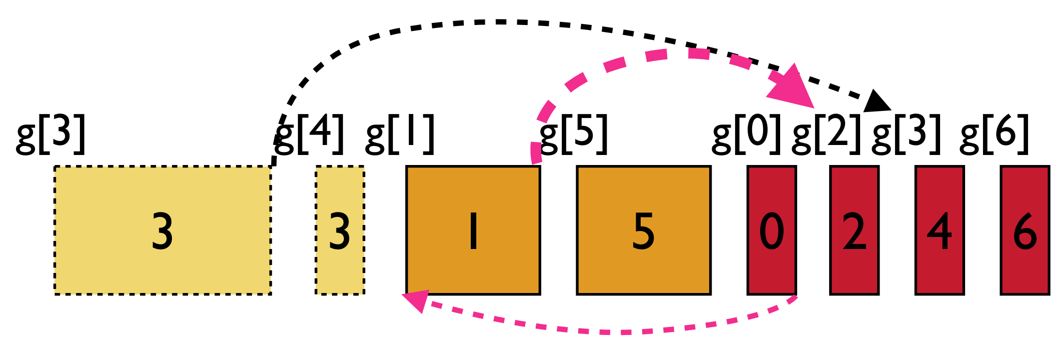
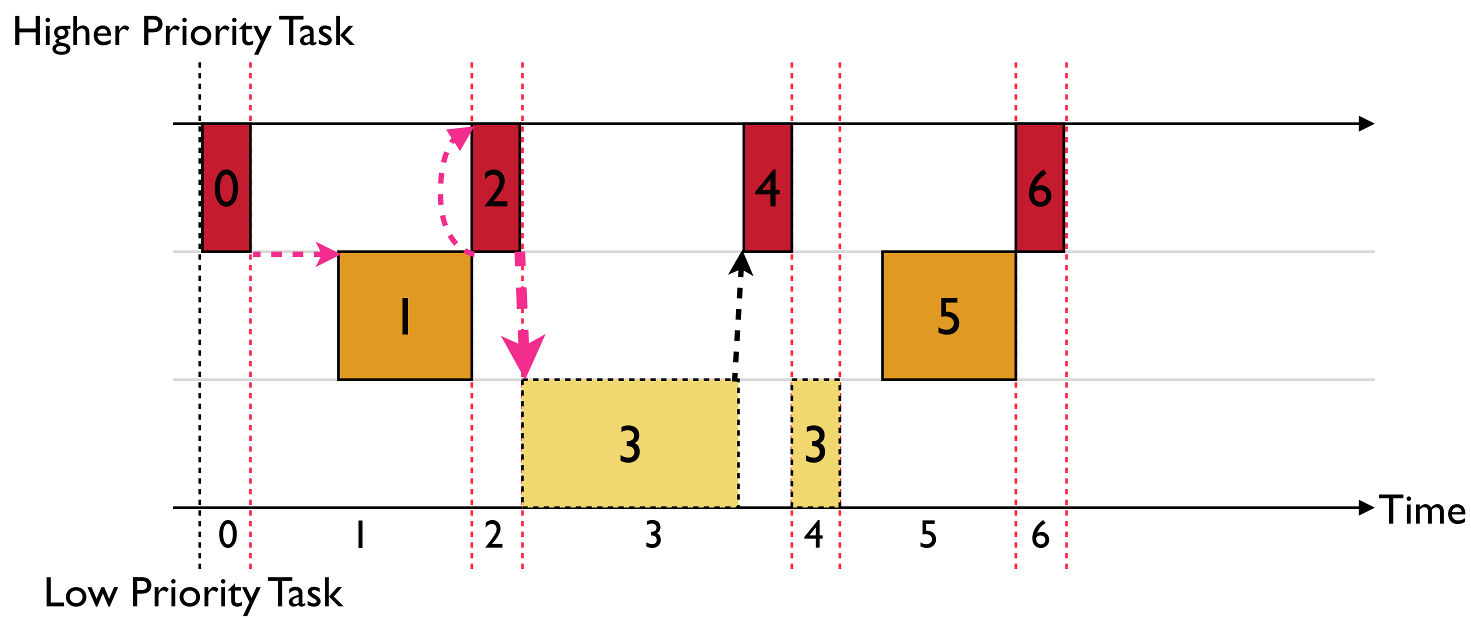


MONOSEQ Sequentialization

Sequential Execution



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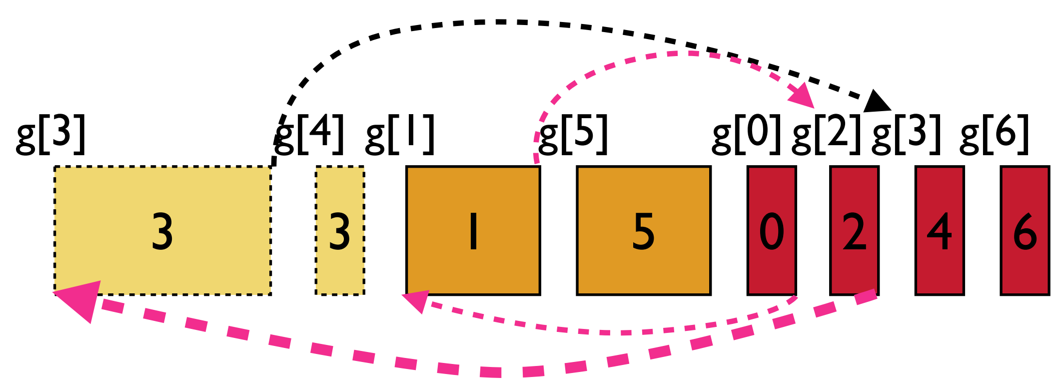
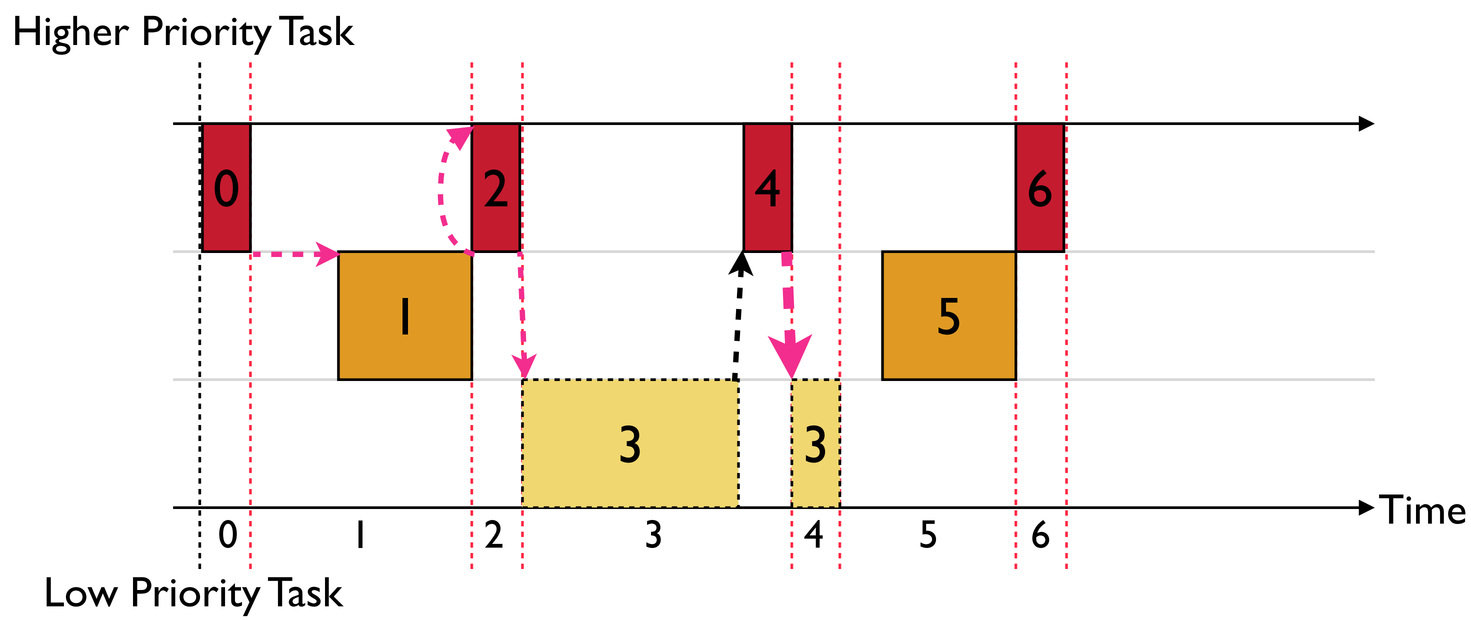


MonoSeq Sequentialization

Sequential Execution



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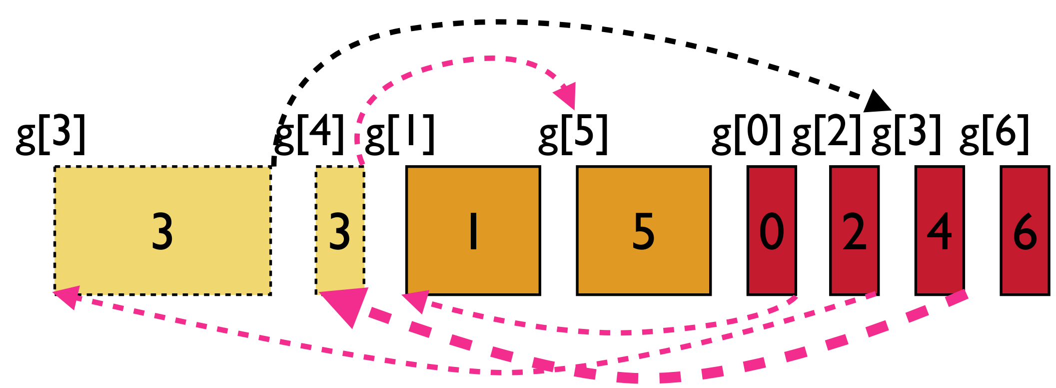
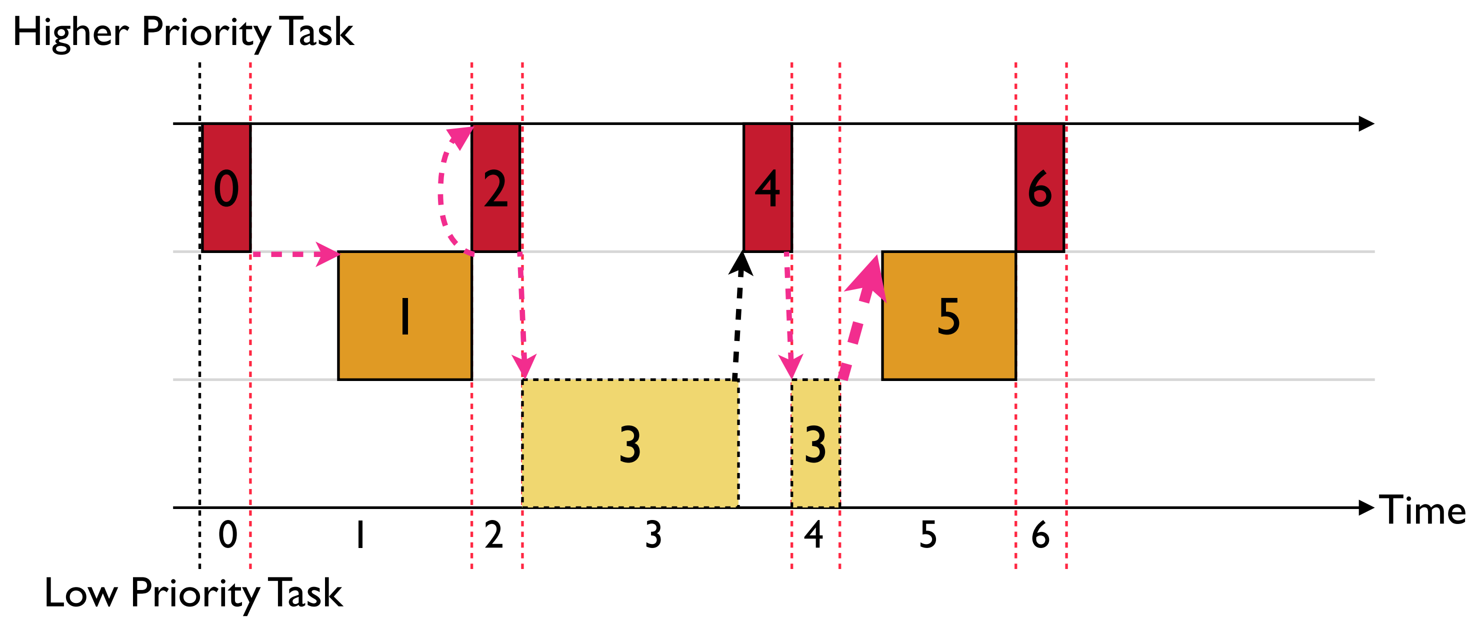


MonoSeq Sequentialization

Sequential Execution



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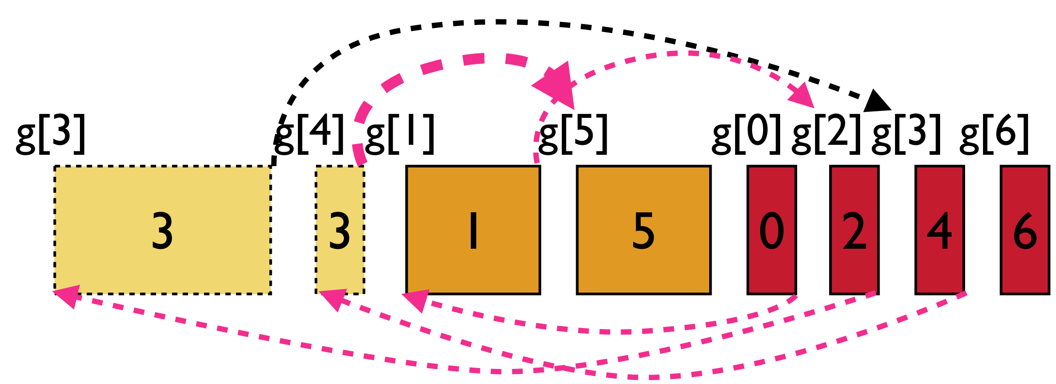
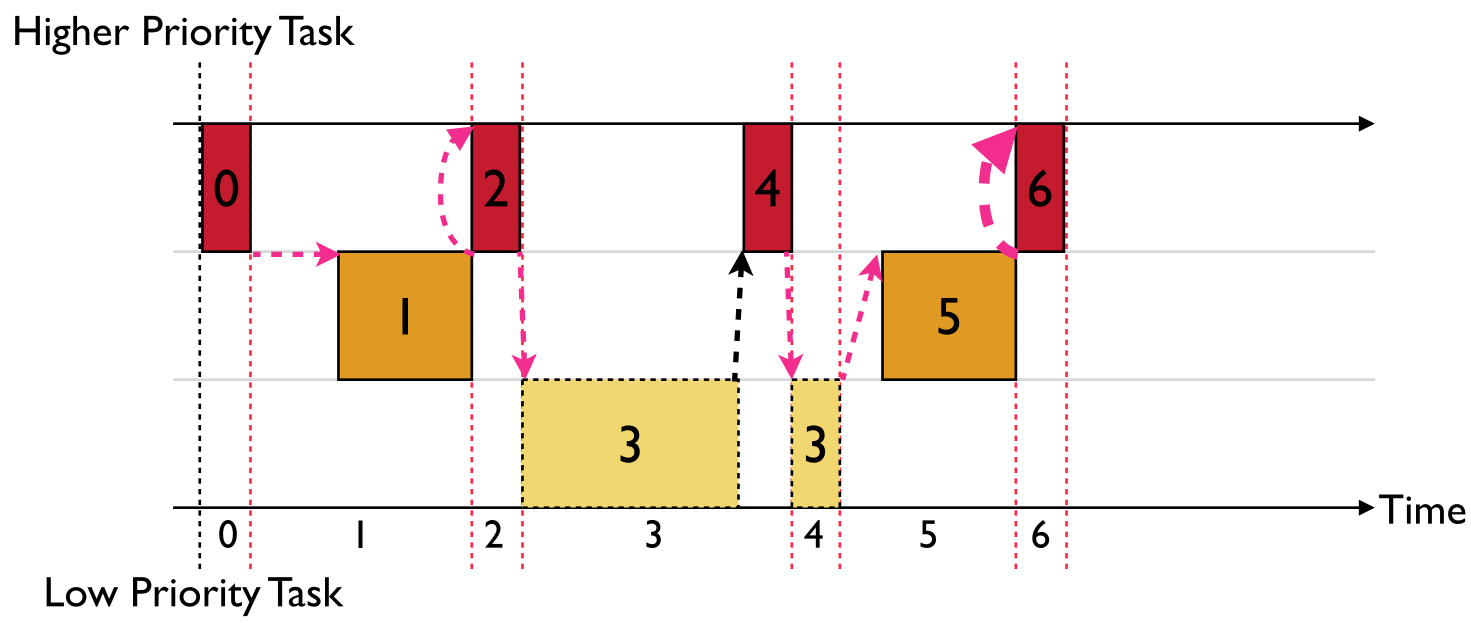


MonoSeq Sequentialization

Sequential Execution



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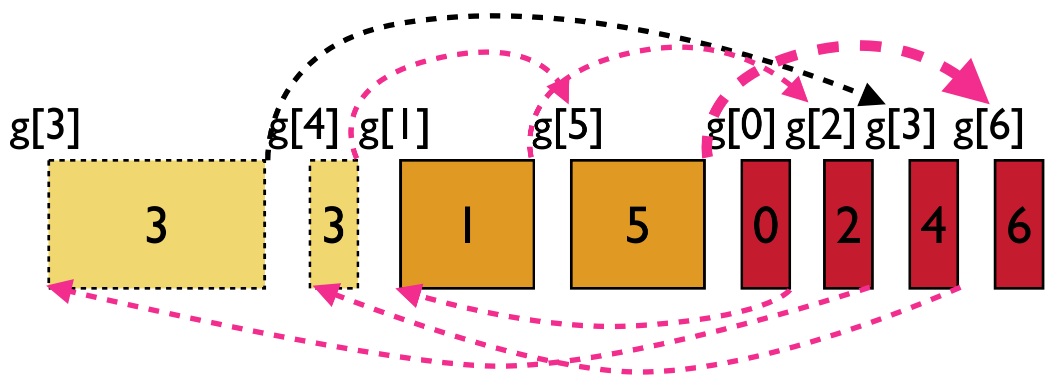
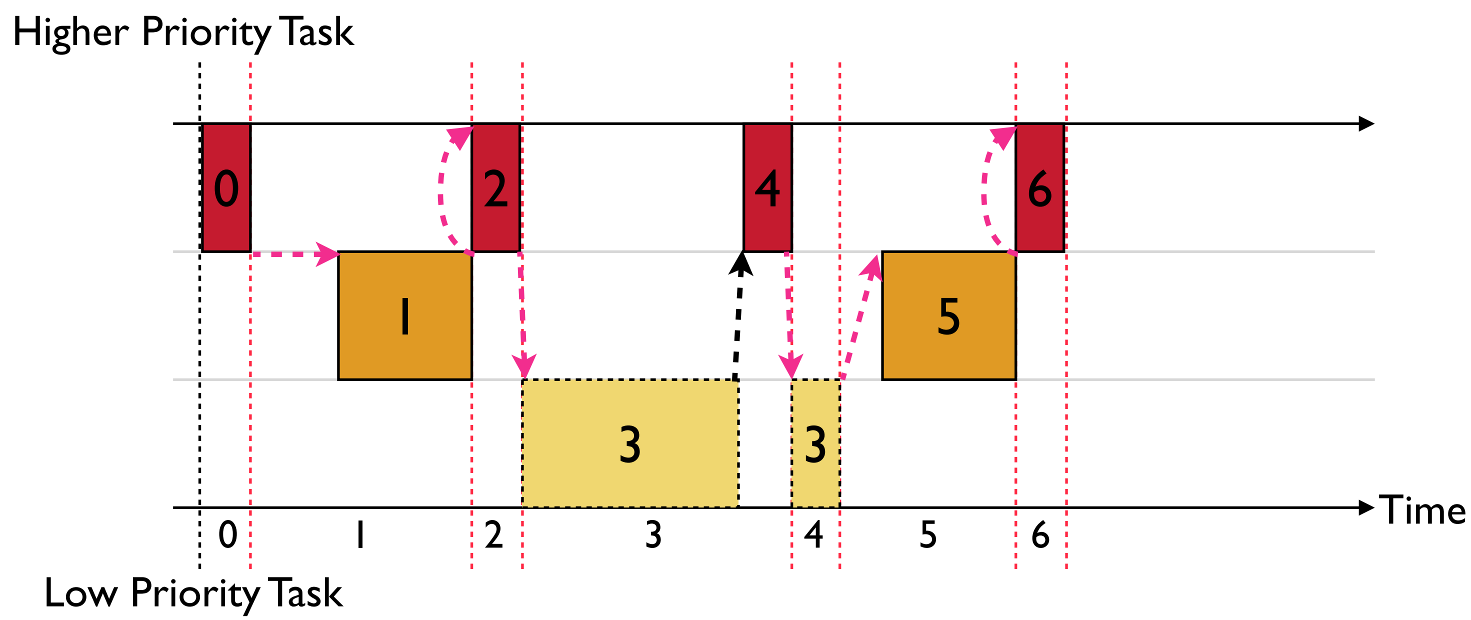
Execute Task Body, from lower priority task to higher priority task

Constraint the value of global variables g[ ]s to respect the order of execution



MonoSeq Sequentialization

Sequential Execution



Guess non-deterministic initial value of each global in each round.

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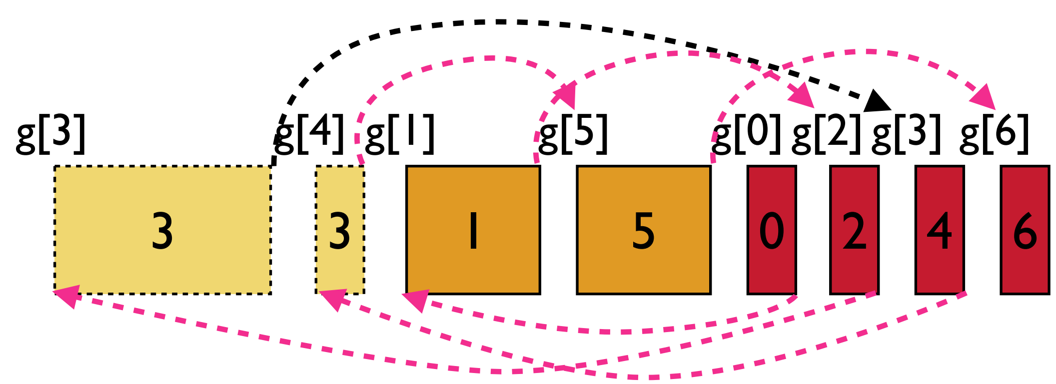

MONOSEQ Sequentialization

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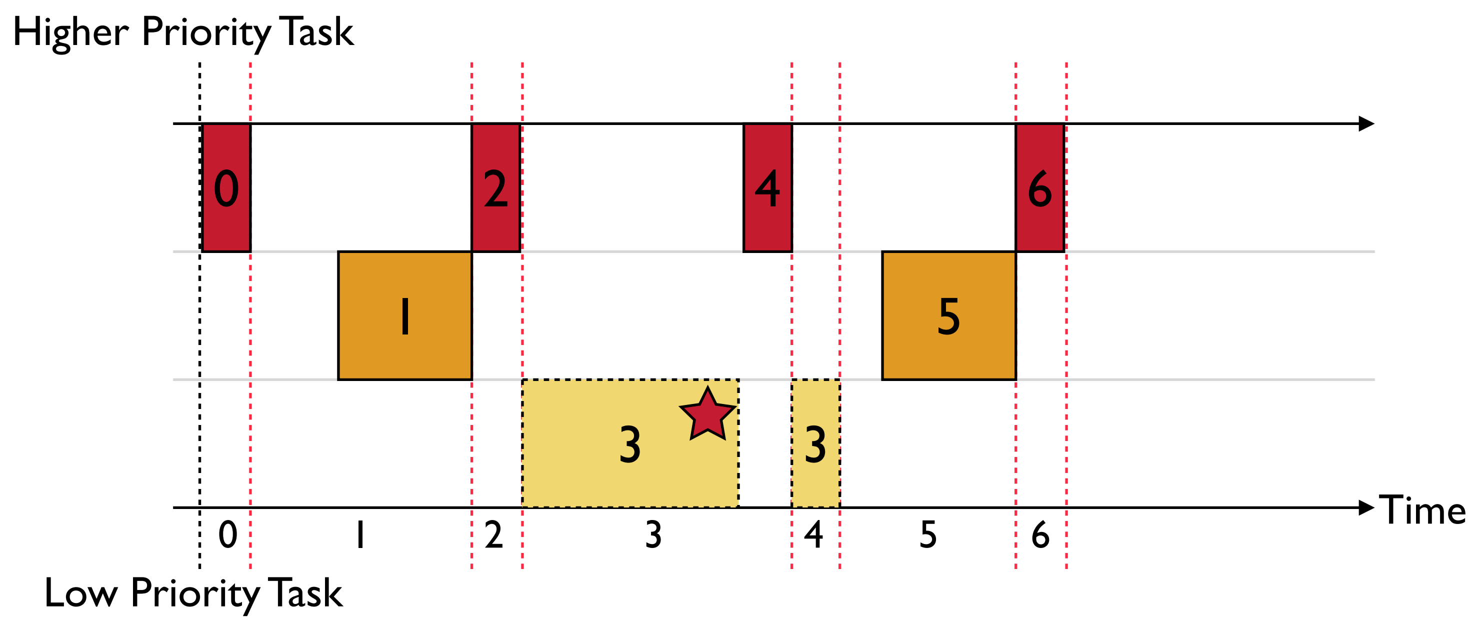

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MonoSeq Sequentialization

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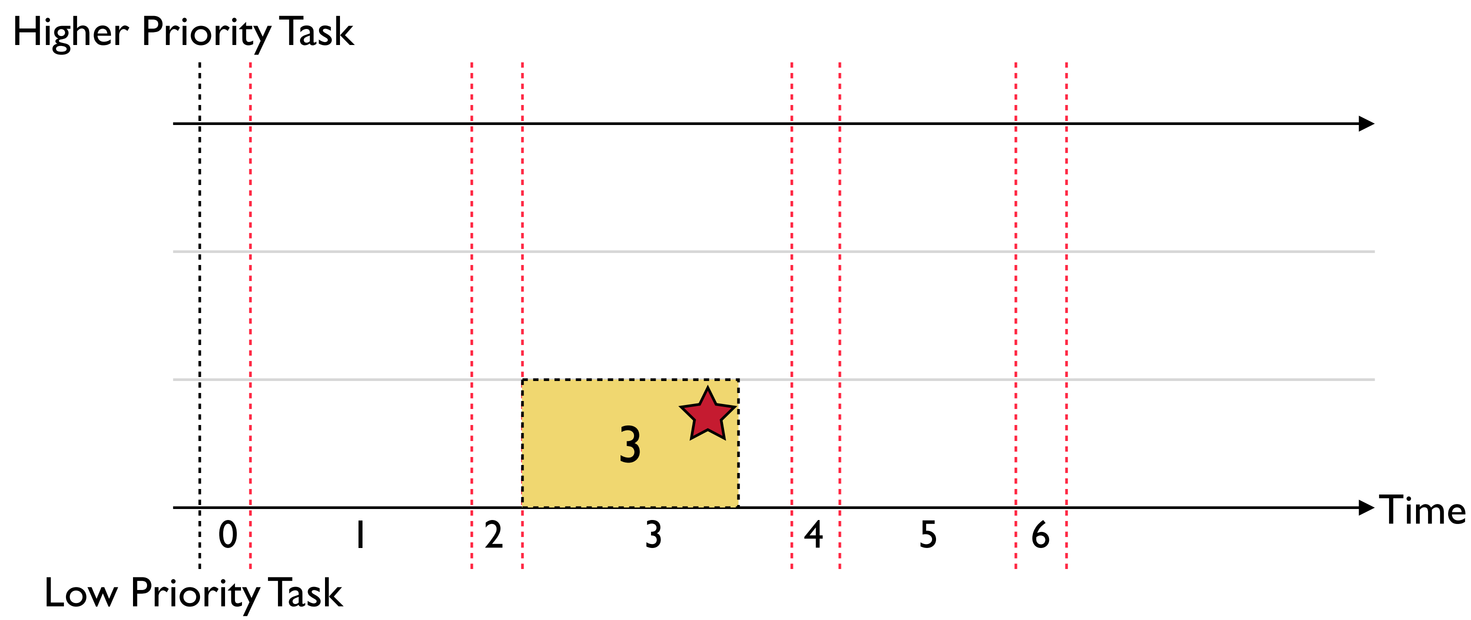

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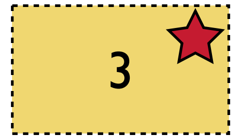

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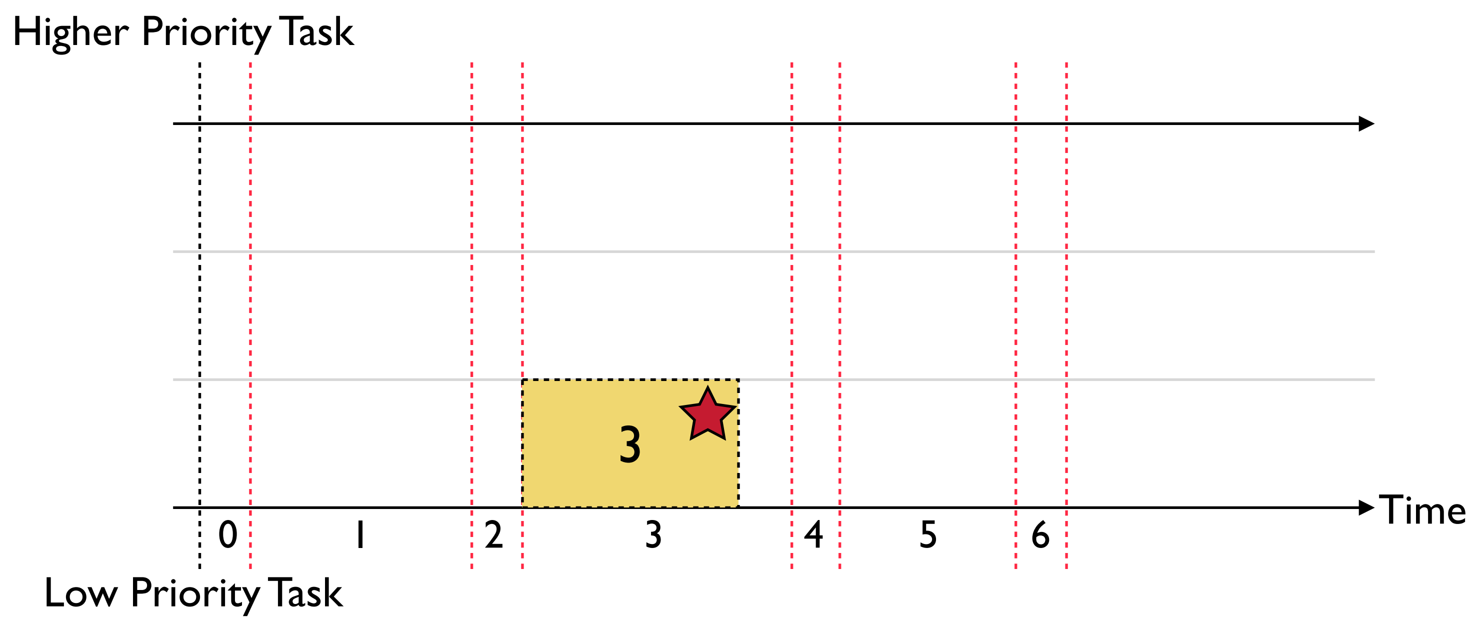

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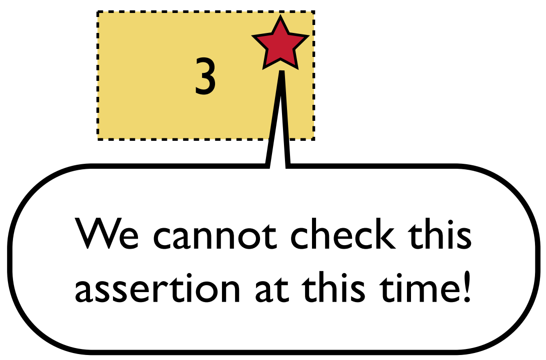

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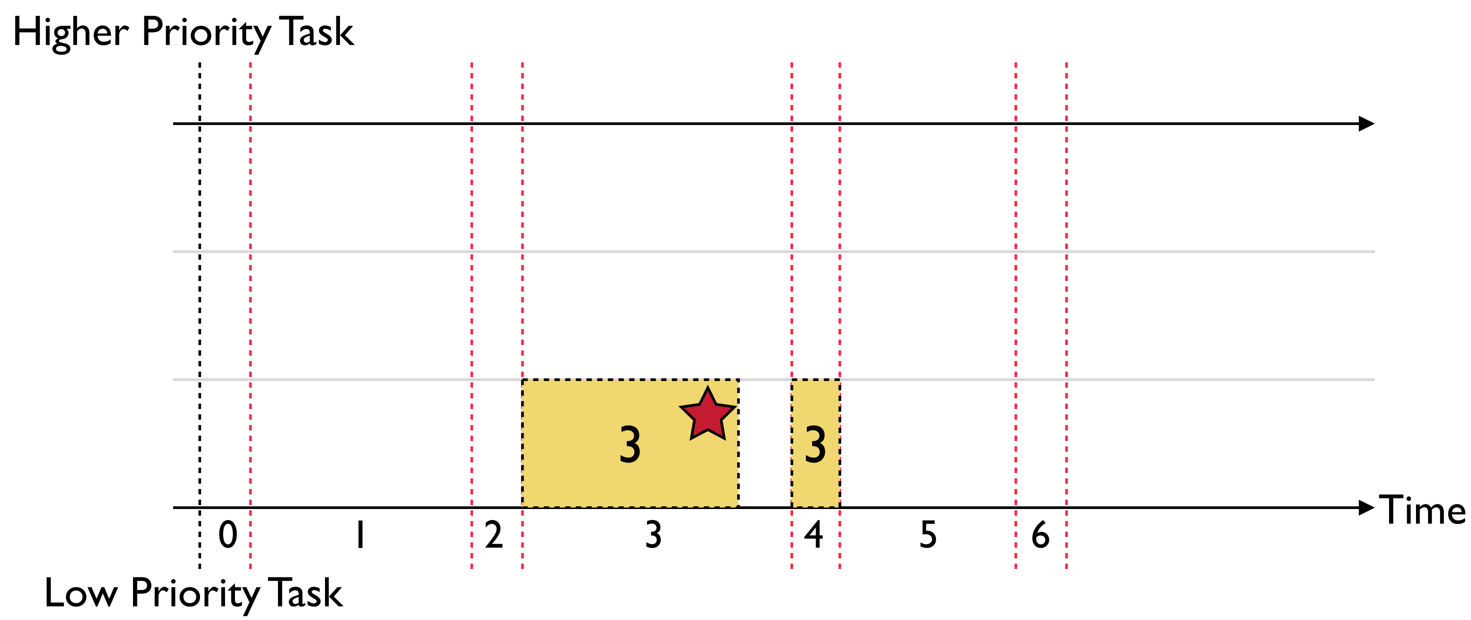

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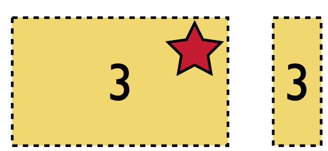

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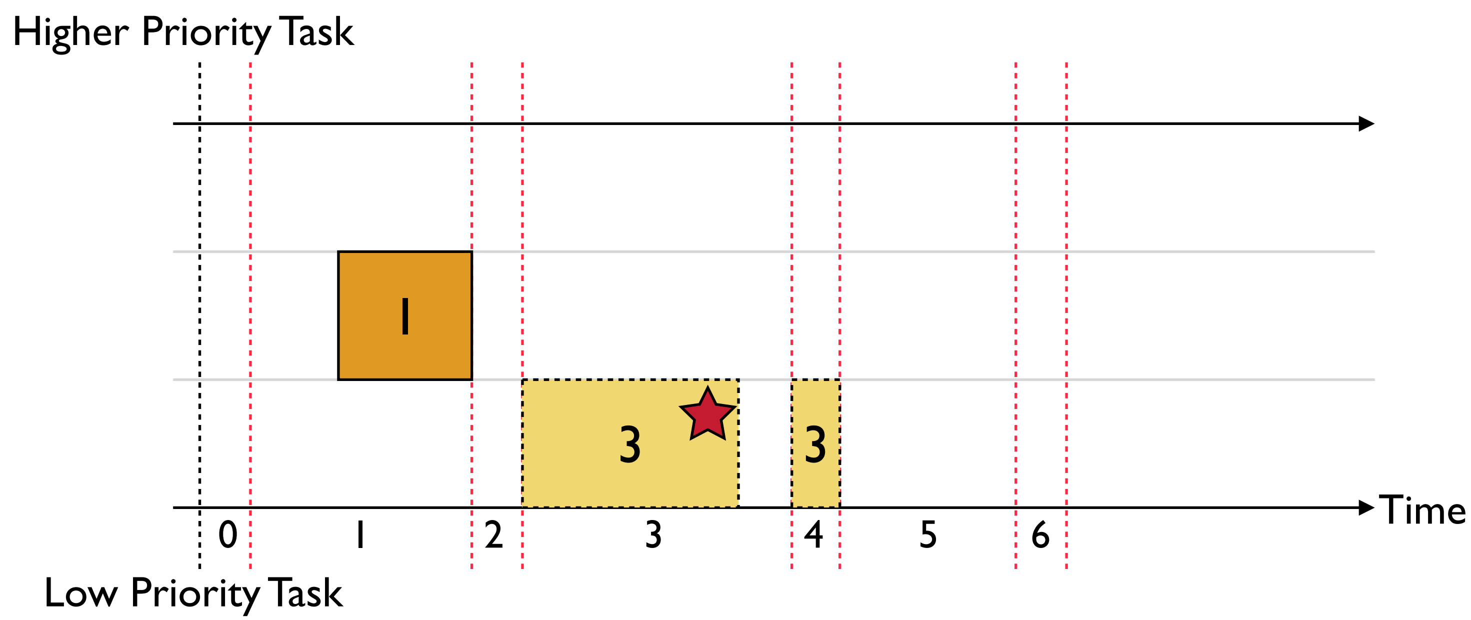

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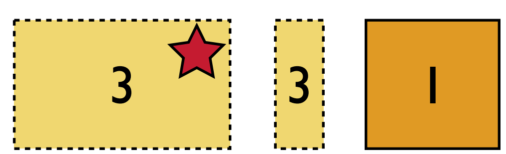

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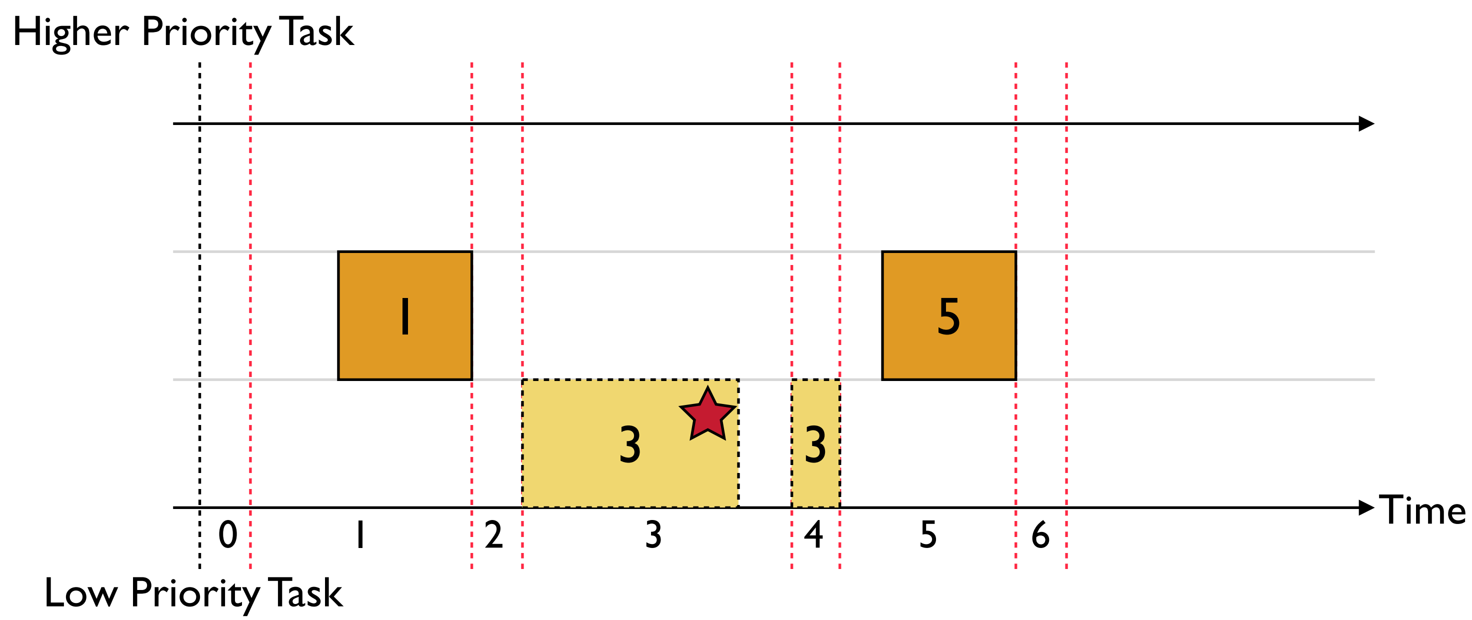

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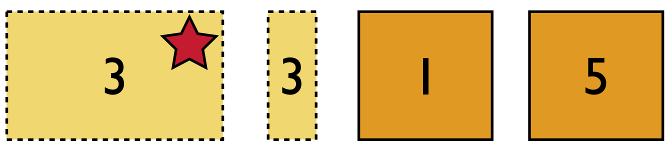

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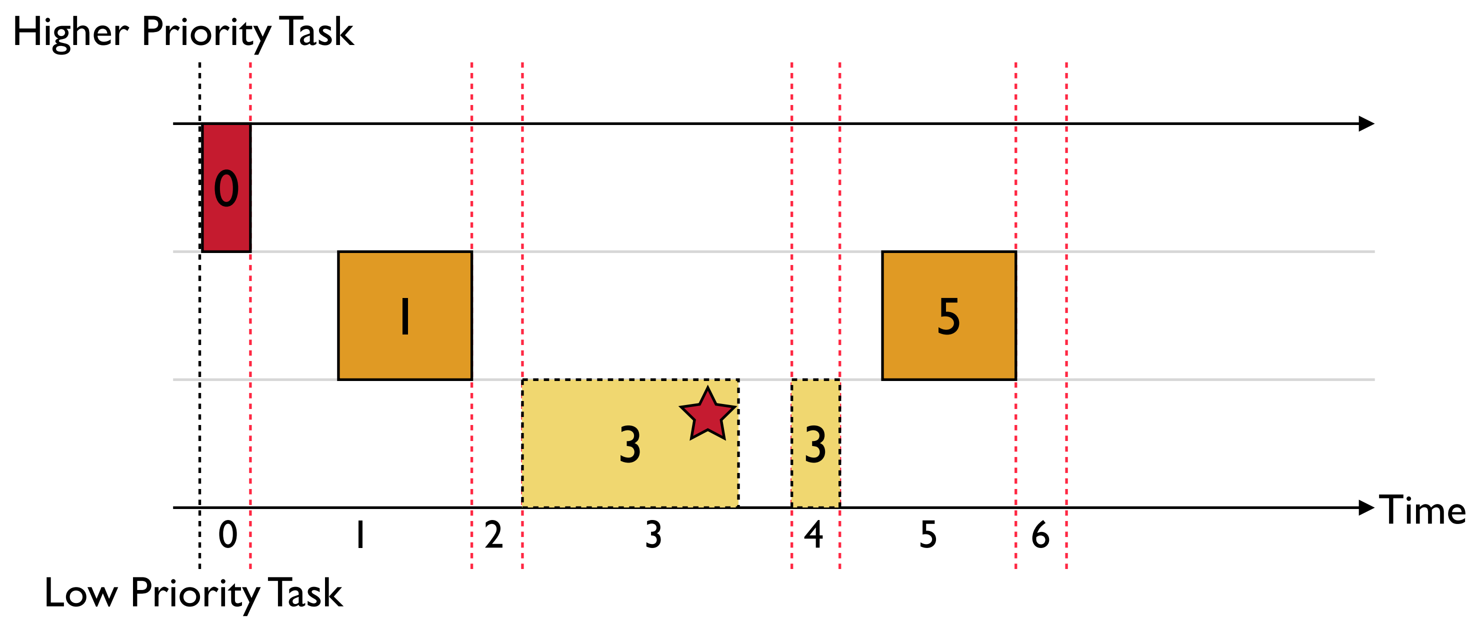

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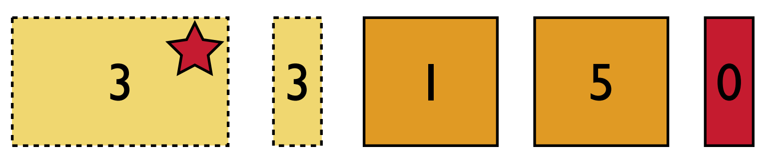

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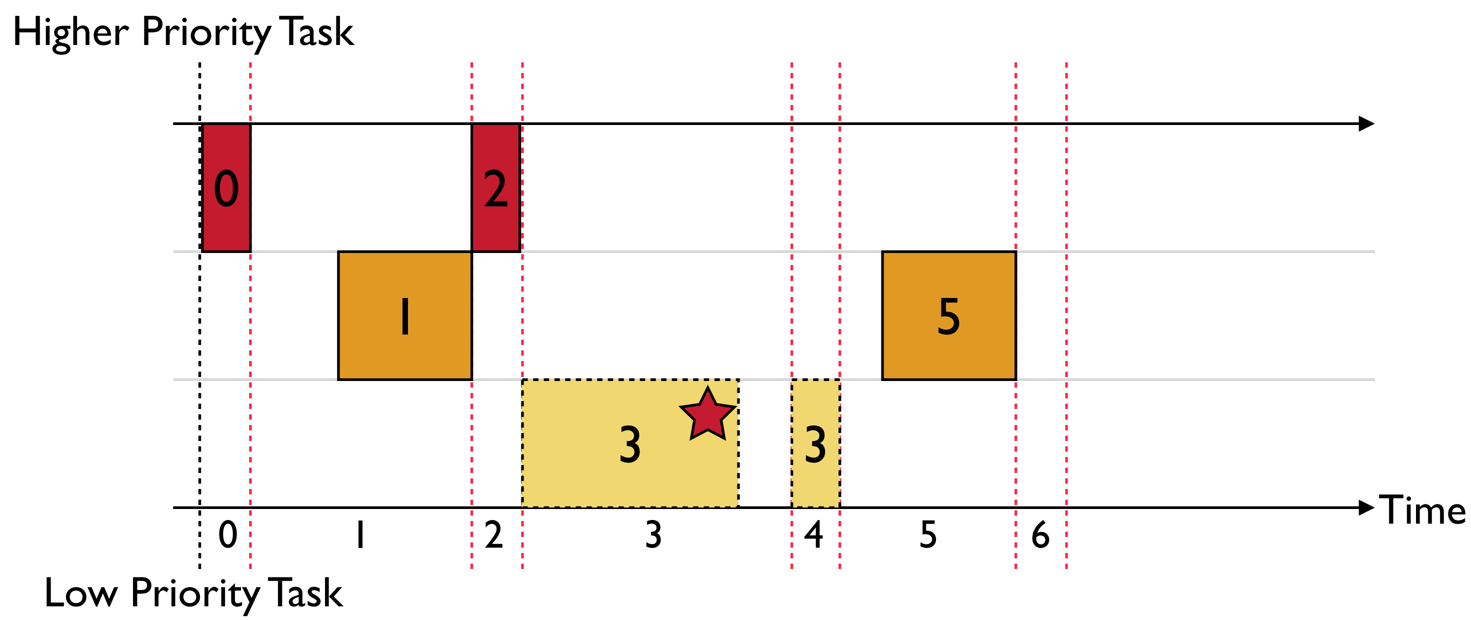

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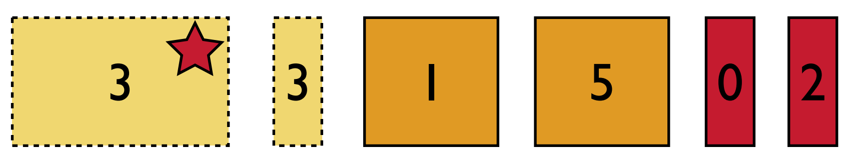

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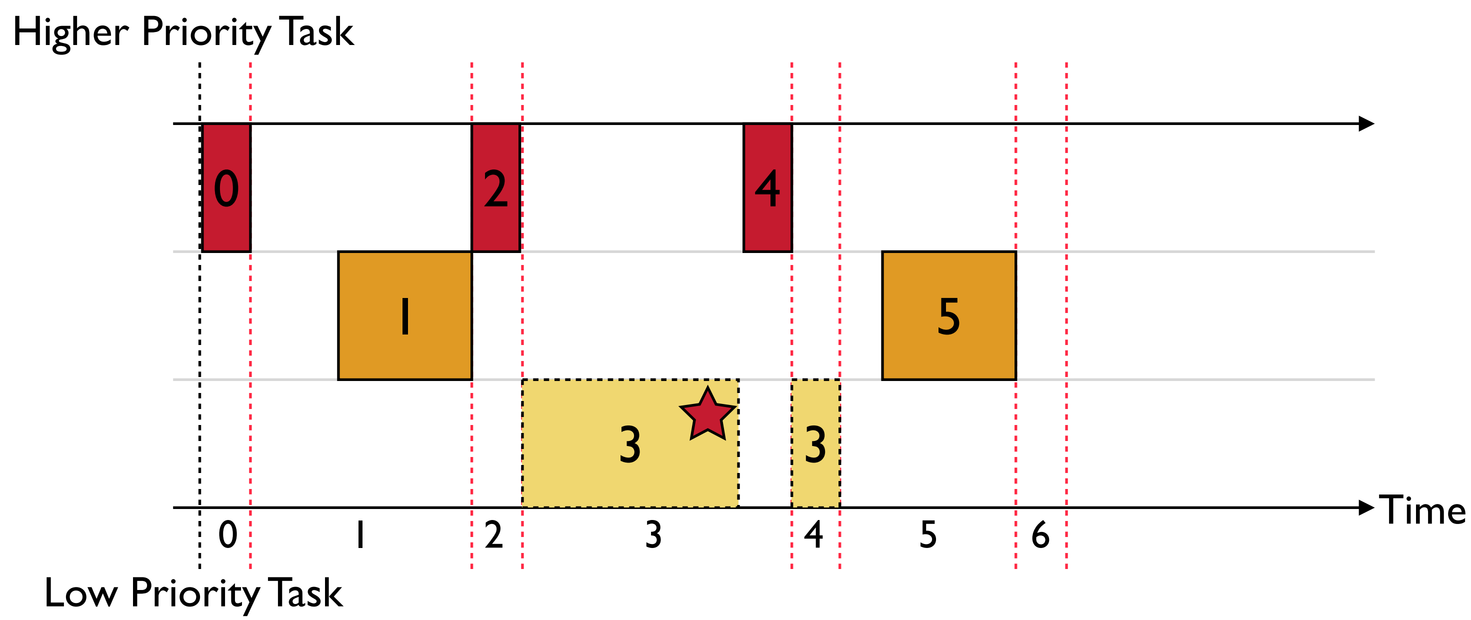

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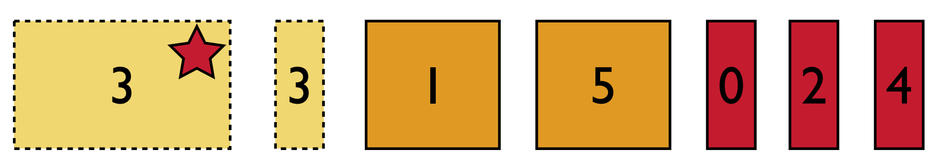

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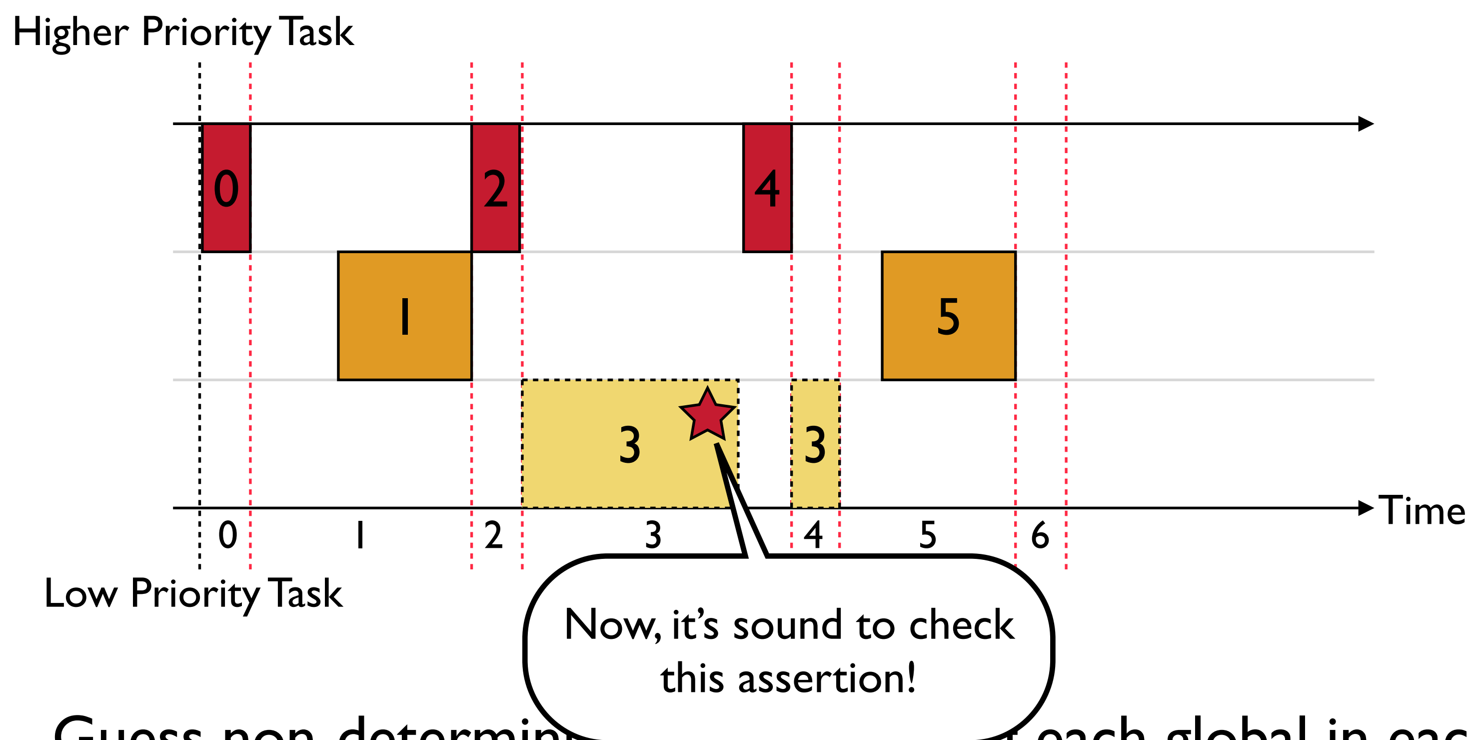

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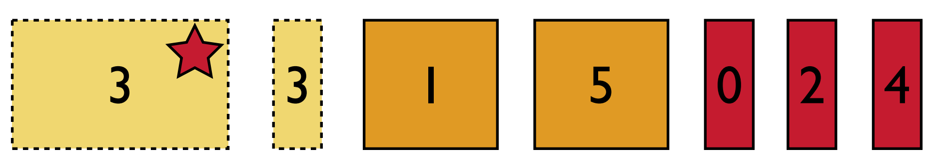

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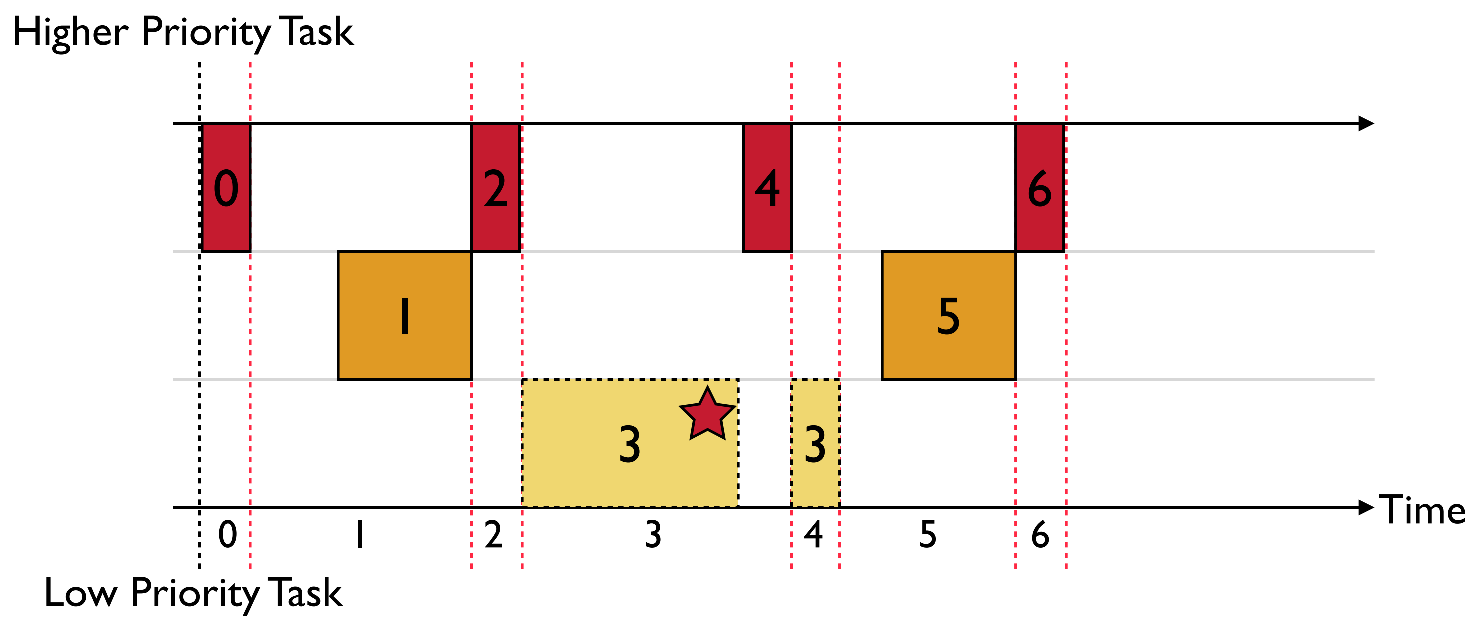

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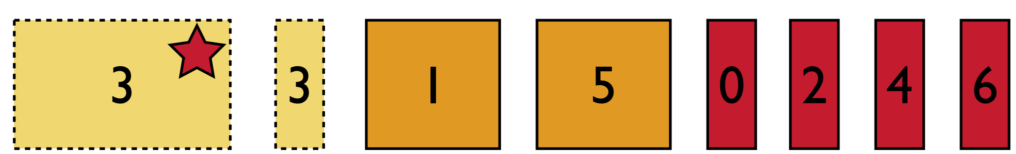

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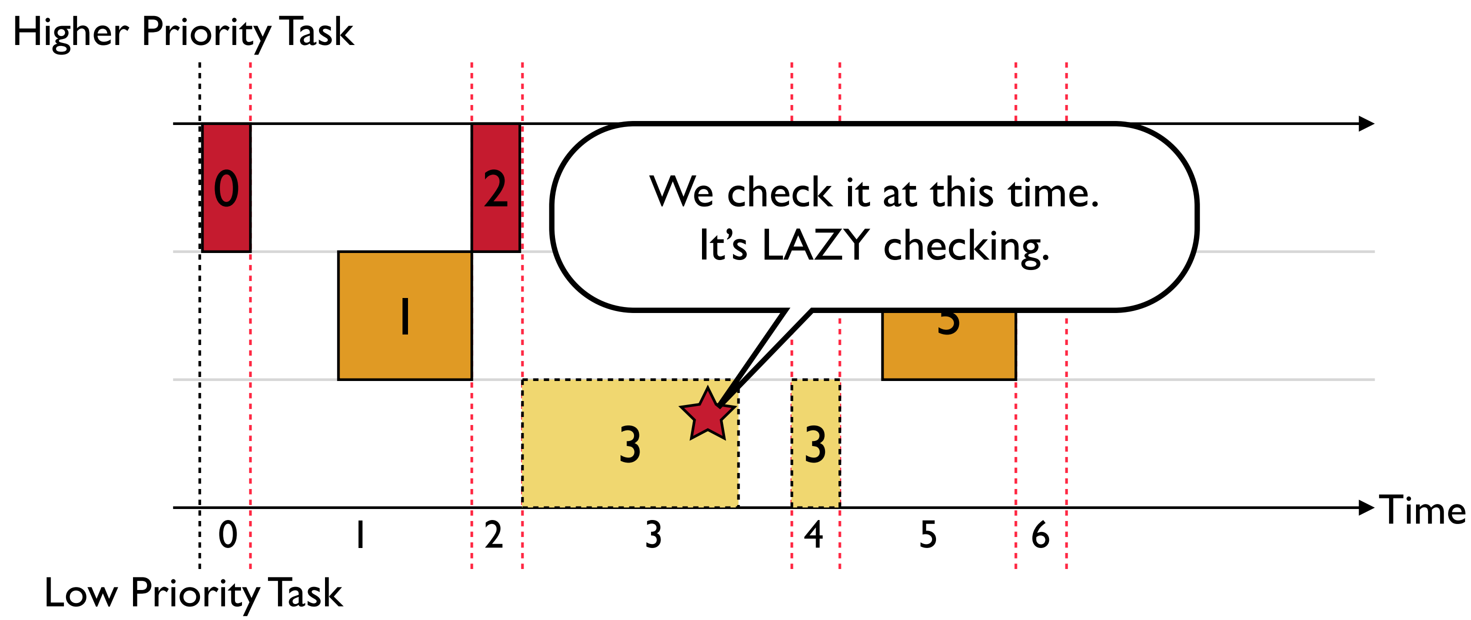

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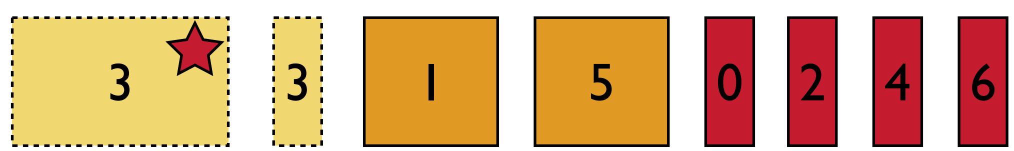

MonoSeq Sequentialization

Algorithm 1 A sequential program $\mathcal{S}$ for a periodic program $\mathcal{C}$ bounded by time $\mathcal{W}$. Notation: T is the set of tasks of $\mathcal{C}$; G is the set of global variables of $\mathcal{C}$; $\mathsf{J}(t)$ is the set of jobs of task t; $R = \sum_{t \in \mathsf{T}, j \in Job(t)} |\mathsf{J}(t)|$ is the number of rounds, $\text{last}(t, j)$ is true iff j is the last job of $t \in \mathsf{T}$; for $t_i, t_j \in \mathsf{T}$, $t_i < t_j$ is true iff t_i is of lower priority than t_j; '$*$' is a non-deterministic value.

var $rnd, job, endRnd, start[\,][\,], end[\,][\,]$
$\forall g \in \mathsf{G} \cdot$ **var** $g[\,], v_g[\,]$
var $localAssert[\,][\,]$

function MAIN( )
  SCHEDULEJOBS()
  $\forall t \in \mathsf{T}, j \in \mathsf{J}(t) \cdot localAssert[t][j] :=$ TRUE
  $\forall g \in \mathsf{G} \cdot g[0] := i_g$
  $\forall g \in \mathsf{G} \forall r \in [1, R) \cdot g[r] := v_g[r]$
  for $t \in \mathsf{T}, job \in \mathsf{J}(t)$ **do**
    $rnd := start[t][job]$
    $endRnd := end[t][job]$
    $\hat{T}_t()$
    assume$(rnd = endRnd)$
  $\forall g \in \mathsf{G}, r \in [0, R-1) \cdot$
   assume$(g[r] = v_g[r+1])$
  $\forall t \in \mathsf{T}, j \in \mathsf{J}(t) \cdot$ assert$(localAssert[t][j])$

function CS(Task t)
  if (*) **then return** FALSE
  $o := rnd$
  $rnd := *$
  assume$(o < rnd \le endRnd)$
  $\forall t' \in \mathsf{T}, j' \in \mathsf{J}(t') \cdot$
   assume(
    $t < t' \Rightarrow$
    $(rnd \le start[t'][j'] \vee rnd > end[t'][j']))$
  return TRUE

function SCHEDULEJOBS( )

  // Jobs are sequential
  $\forall t \in \mathsf{T}, j \in \mathsf{J}(t) \cdot$
   assume(
    $0 \le start[t][j] \le end[t][j] \le R \wedge$
    $(\neg \text{last}(t, j) \Rightarrow end[t][j] \le start[t][j+1]))$

  // Jobs are well-nested
  $\forall t_1 \in \mathsf{T}, t_2 \in \mathsf{T}, j_1 \in \mathsf{J}(t_1), j_2 \in \mathsf{J}(t_2) \cdot$
   assume(
    $\big(t_1 < t_2 \wedge$
    $start[t_1][j_1] \le end[t_2][j_2] \wedge$
    $start[t_2][j_2] \le end[t_1][j_1]\big) \Rightarrow$
     $(start[t_1][j_1] \le start[t_2][j_2] \le end[t_2][j_2] < end[t_1][j_1]))$

  // Jobs respect preemption bounds
  $\forall t_1 \in \mathsf{T}, t_2 \in \mathsf{T}, j_1 \in \mathsf{J}(t_1), j_2 \in \mathsf{J}(t_2) \cdot$
   assume(
    $\big(t_1 < t_2 \wedge j_2 \ge PB_{t_1}^{t_2} \wedge$
    $start[t_1][j_1] \le start[t_2][j_2] \le end[t_2][j_2] < end[t_1][j_1]\big) \Rightarrow$
     $end[t_2][j_2 - PB_{t_1}^{t_2}] < start[t_1][j_1])$

function $\hat{T}_t$( )
  Same as T_t, but
   each statement 'st' is replaced with:
    CS(t) ; $st[g \leftarrow g[rnd]]$,
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MonoSeq

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  $\forall t \in \mathsf{T}, j \in \mathsf{J}(t) \cdot$
   assume(
    $0 \le start[t][j] \le end[t][j] \le R \wedge$
    $(\neg\text{last}(t, j) \Rightarrow end[t][j] \le start[t][j+1]))$

// Jobs are well-nested
  $\forall t_1 \in \mathsf{T}, t_2 \in \mathsf{T}, j_1 \in \mathsf{J}(t_1), j_2 \in \mathsf{J}(t_2) \cdot$
   assume(
    $(t_1 < t_2 \wedge$
    $start[t_1][j_1] \le end[t_2][j_2] \wedge$
    $start[t_2][j_2] \le end[t_1][j_1]) \Rightarrow$
    $(start[t_1][j_1] \le start[t_2][j_2] \le end[t_2][j_2] < end[t_1][j_1]))$

// Jobs respect preemption bounds
  $\forall t_1 \in \mathsf{T}, t_2 \in \mathsf{T}, j_1 \in \mathsf{J}(t_1), j_2 \in \mathsf{J}(t_2) \cdot$
   assume(
    $(t_1 < t_2 \wedge j_2 \ge PB_{t_1}^{t_2} \wedge$
    $start[t_1][j_1] \le start[t_2][j_2] \le end[t_2][j_2] < end[t_1][j_1]) \Rightarrow$
    $end[t_2][j_2 - PB_{t_1}^{t_2}] < start[t_1][j_1])$

function $\hat{T}_t$( )
Same as T_t, but
 each statement 'st' is replaced with:
  CS(t) ; $st[g \leftarrow g[rnd]]$,
 and each 'assert(e)' is replaced with:
  $localAssert[t][job] := e$

MonoSeq

Algorithm 1 A sequential program $\mathcal{S}$ for a periodic program $\mathcal{C}$ bound... by time $\mathcal{W}$. Notation: T is the set of tasks of $\mathcal{C}$; G is the set of global variables of $\mathcal{C}$; $\mathsf{J}(t)$ is the set of ... is true iff j is the last job of $t \in \mathsf{T}$; for $t_i, t_j \in \mathsf{T}$, ... value.

It also supports two types of common locking mechanisms by encoding them as constraints:

1. Preemption Lock
2. Priority Ceiling Lock

var $rnd, job, endRnd, start[\,][\,], end[\,][\,]$
$\forall g \in \mathsf{G} \cdot$ **var** $g[\,], v_g[\,]$
var *localAssert*$[\,][\,]$

function MAIN( )
  SCHEDULEJOBS()
  $\forall t \in \mathsf{T}, j \in \mathsf{J}(t) \cdot$ *localAssert*$[t][j] :=$ TRUE
  $\forall g \in \mathsf{G} \cdot g[0] := i_g$
  $\forall g \in \mathsf{G} \forall r \in [1, R) \cdot g[r] := v_g[r]$
  for $t \in \mathsf{T}, job \in \mathsf{J}(t)$ **do**
    $rnd := start[t][job]$
    $endRnd := end[t][job]$
    $\hat{T}_t()$
    assume$(rnd = endRnd)$
  $\forall g \in \mathsf{G}, r \in [0, R-1) \cdot$ assume$(g[r] = v_g[r+1])$
  $\forall t \in \mathsf{T}, j \in \mathsf{J}(t) \cdot$ assert$(localAssert[t][j])$

function CS(Task t)
  if (*) **then return** FALSE
  $o := rnd$
  $rnd := *$
  assume$(o < rnd \le endRnd)$
  $\forall t' \in \mathsf{T}, j' \in \mathsf{J}(t') \cdot$ assume($t < t' \Rightarrow (rnd \le start[t'][j'] \vee rnd > end[t'][j']))$
  return TRUE

$(\neg \text{last}(t, j) \Rightarrow end[t][j] \le start[t][j+1]))$

// Jobs are well-nested

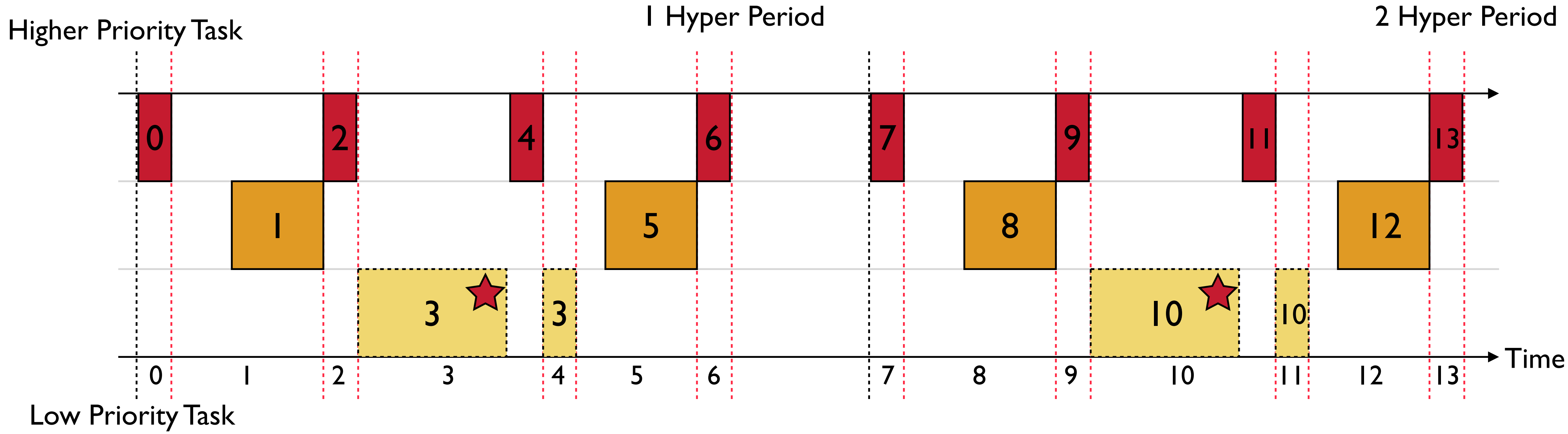
$\forall t_1 \in \mathsf{T}, t_2 \in \mathsf{T}, j_1 \in \mathsf{J}(t_1), j_2 \in \mathsf{J}(t_2) \cdot$ assume($\big(t_1 < t_2 \wedge start[t_1][j_1] \le end[t_2][j_2] \wedge start[t_2][j_2] \le end[t_1][j_1]\big) \Rightarrow (start[t_1][j_1] \le start[t_2][j_2] \le end[t_2][j_2] < end[t_1][j_1]))$

// Jobs respect preemption bounds

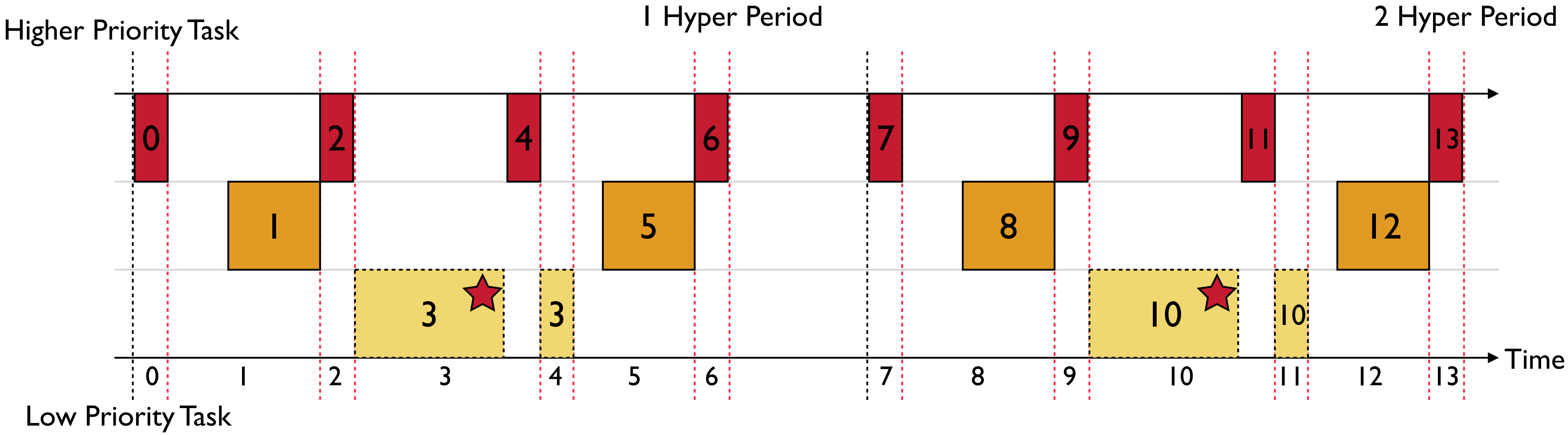
$\forall t_1 \in \mathsf{T}, t_2 \in \mathsf{T}, j_1 \in \mathsf{J}(t_1), j_2 \in \mathsf{J}(t_2) \cdot$ assume($\big(t_1 < t_2 \wedge j_2 \ge PB_{t_1}^{t_2} \wedge start[t_1][j_1] \le start[t_2][j_2] \le end[t_2][j_2] < end[t_1][j_1]\big) \Rightarrow end[t_2][j_2 - PB_{t_1}^{t_2}] < start[t_1][j_1])$

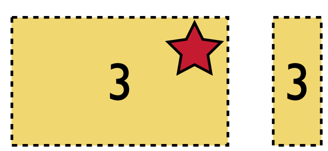
function $\hat{T}_t$( )
  Same as T_t, but each statement 'st' is replaced with:
    CS(t) ; $st[g \leftarrow g[rnd]]$,
  and each 'assert(e)' is replaced with:
    localAssert$[t][job] := e$

Observations



Observations





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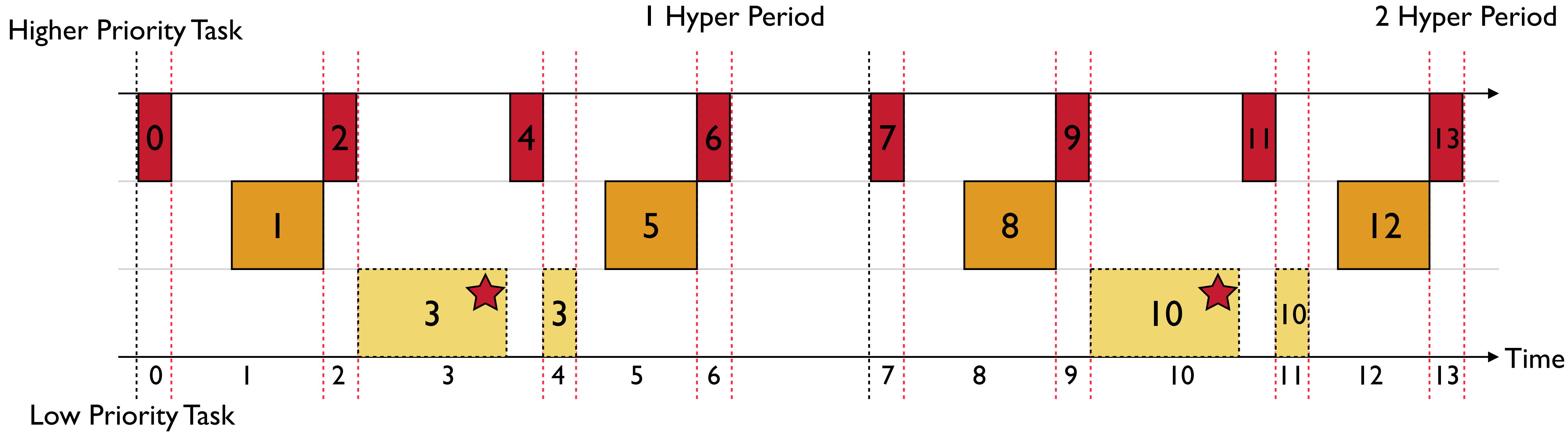

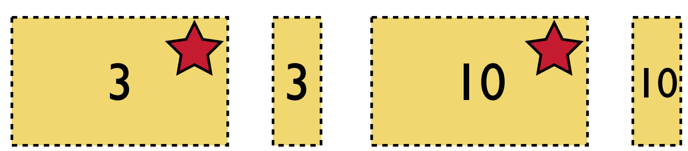

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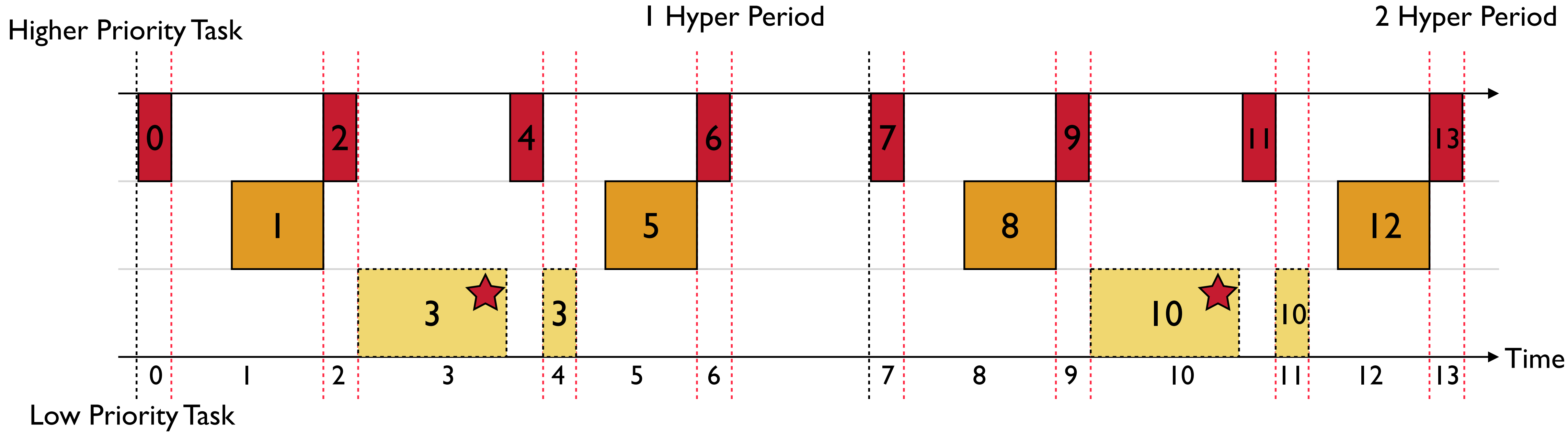

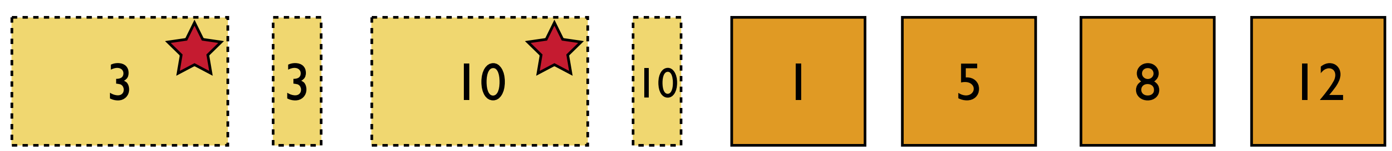

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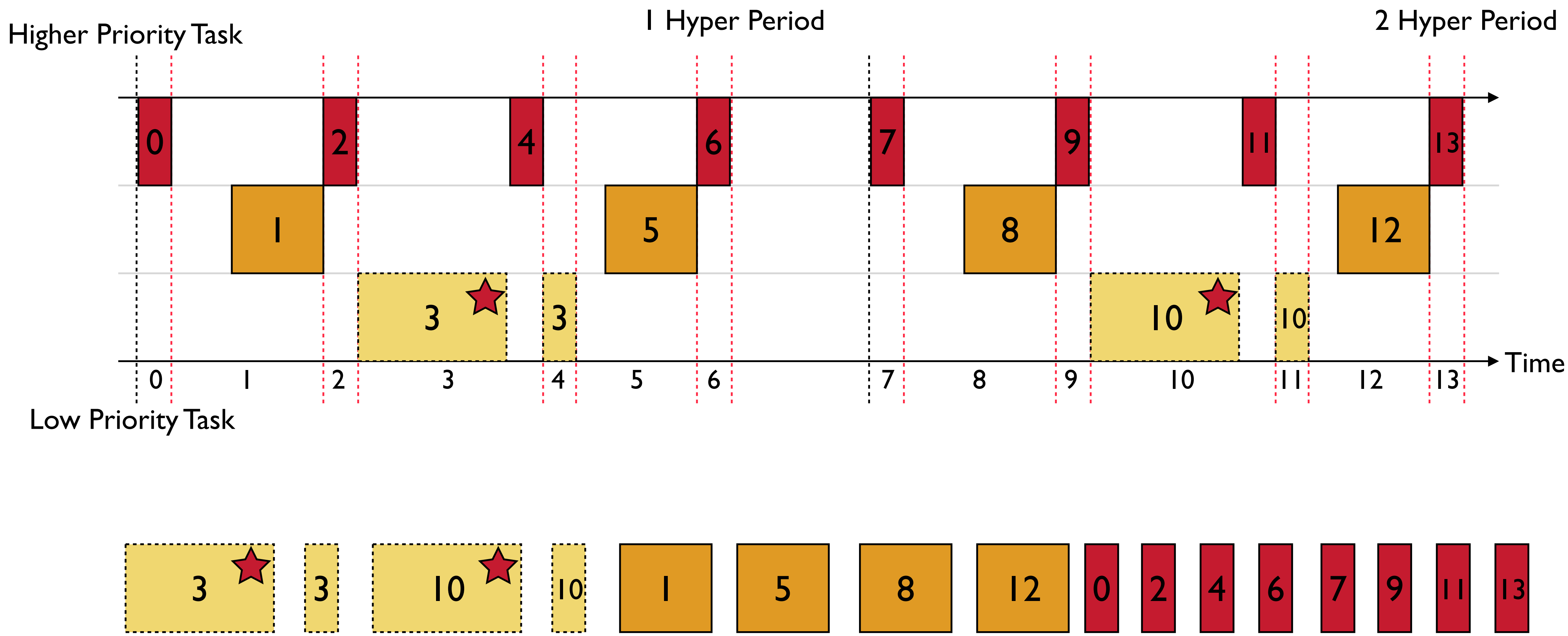

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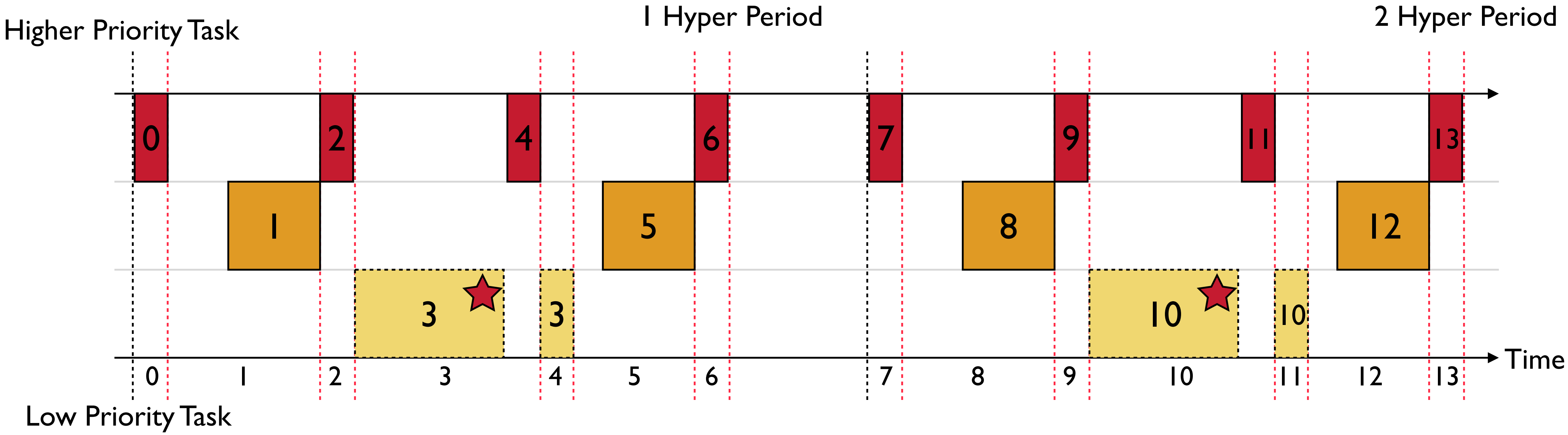

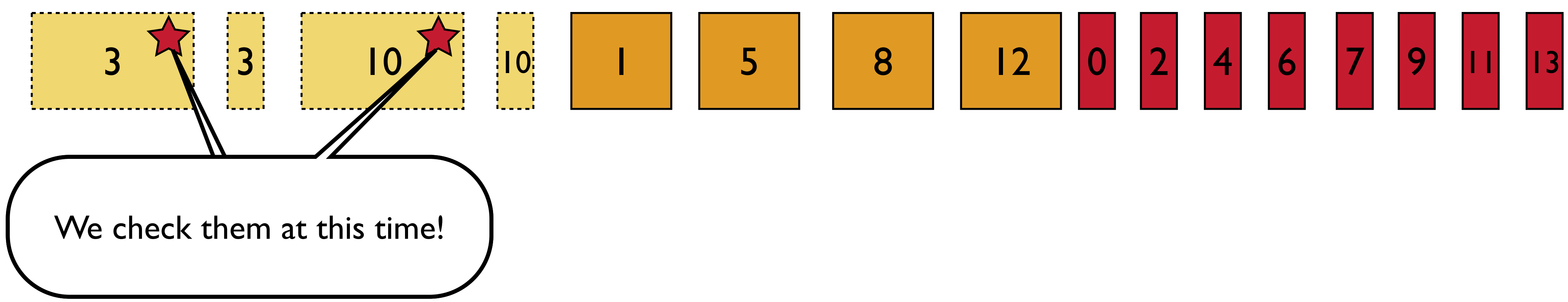

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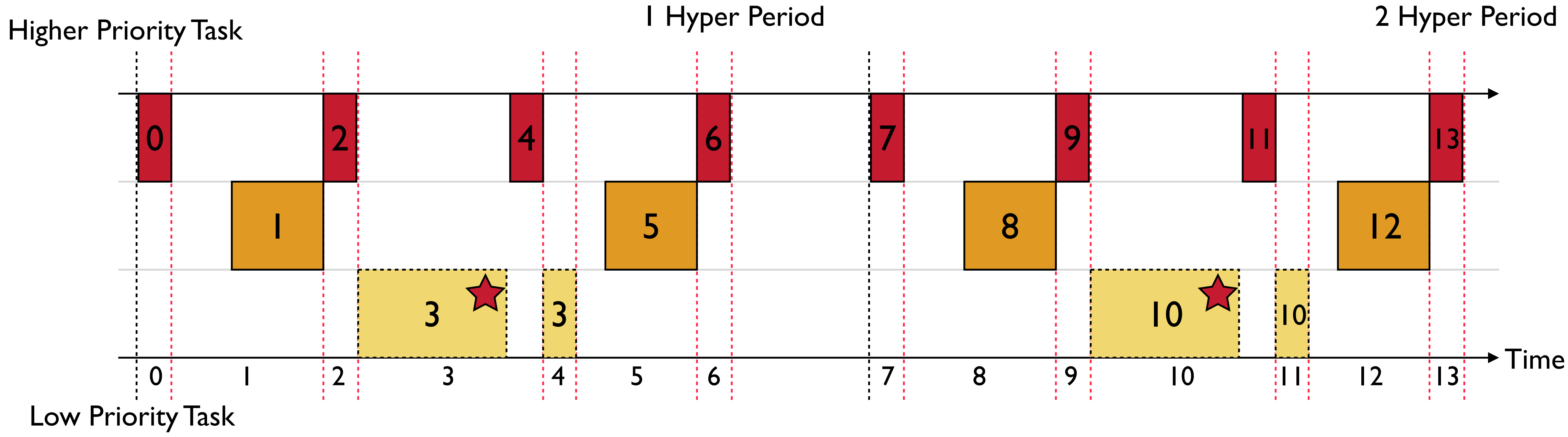

We should consider "Hyper-period"!

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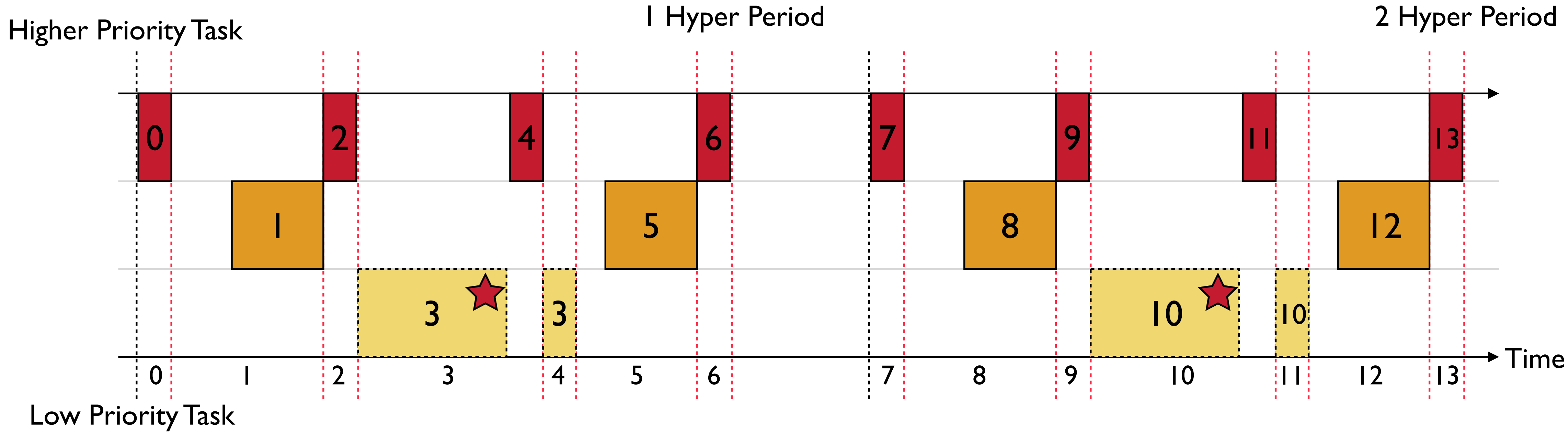

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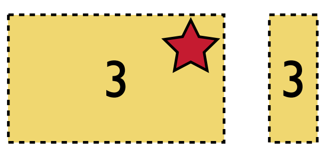

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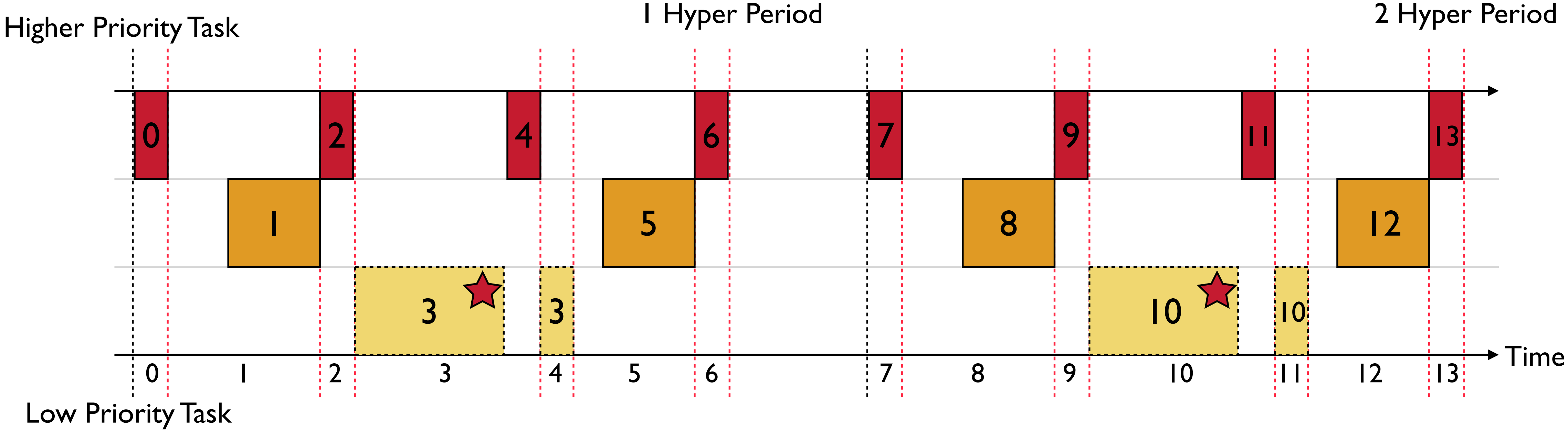

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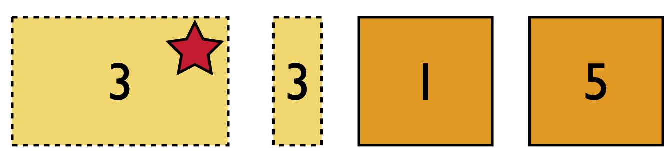

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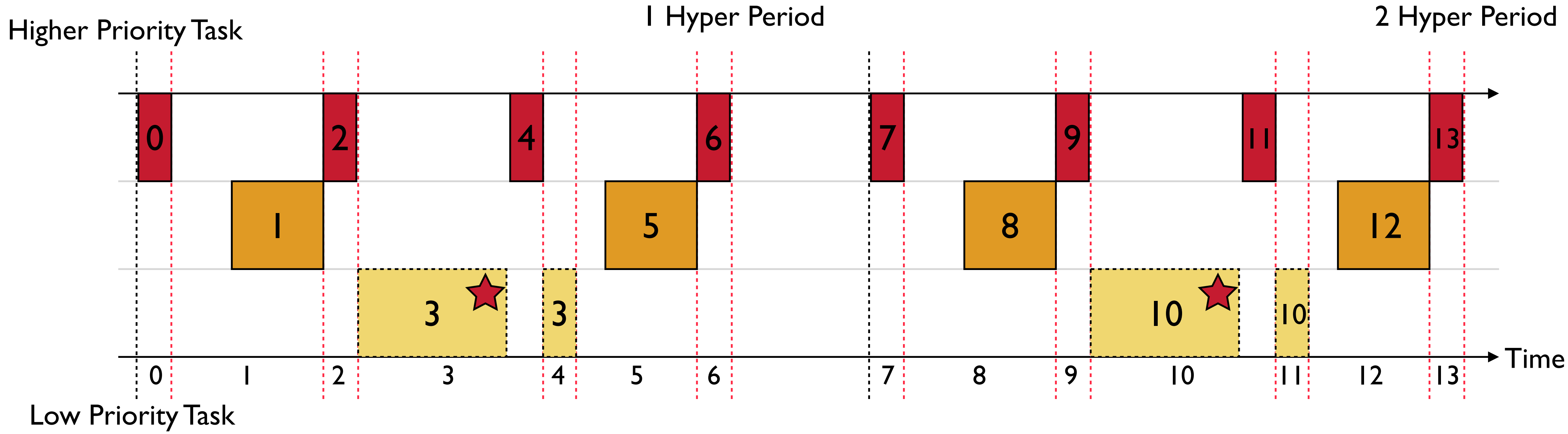

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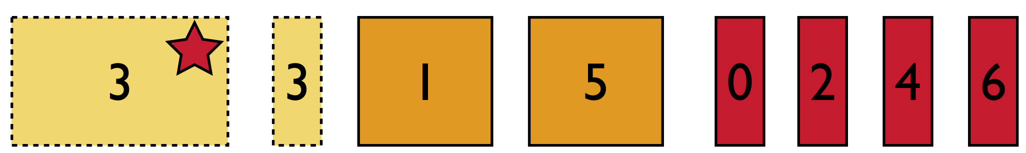

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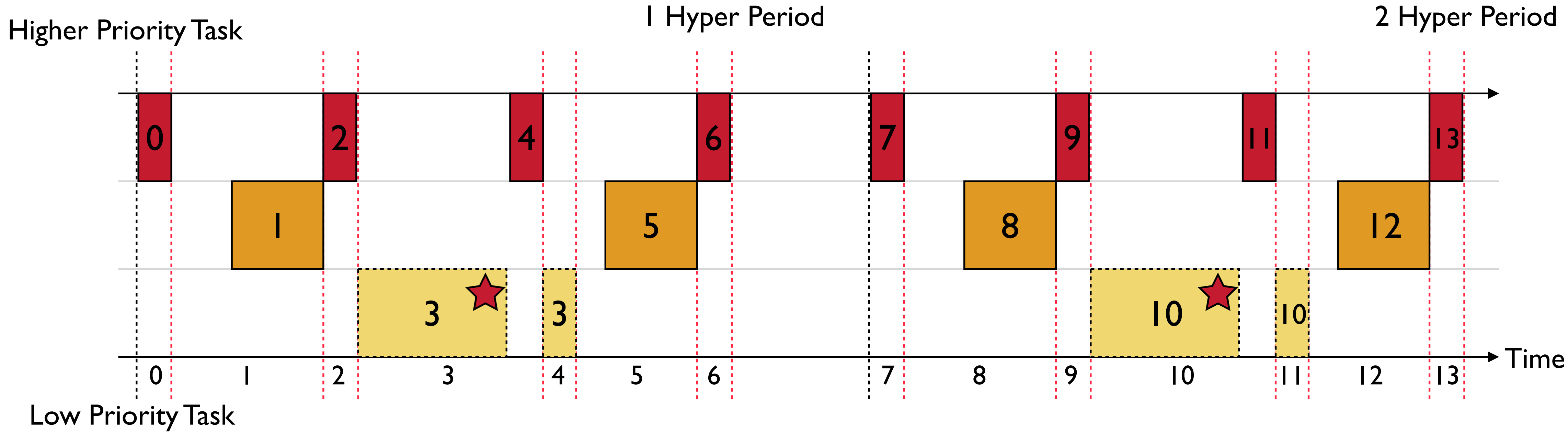

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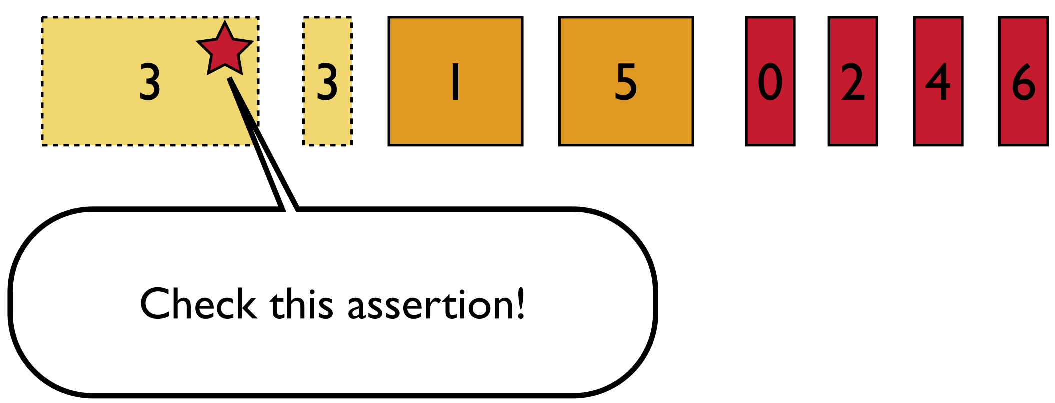

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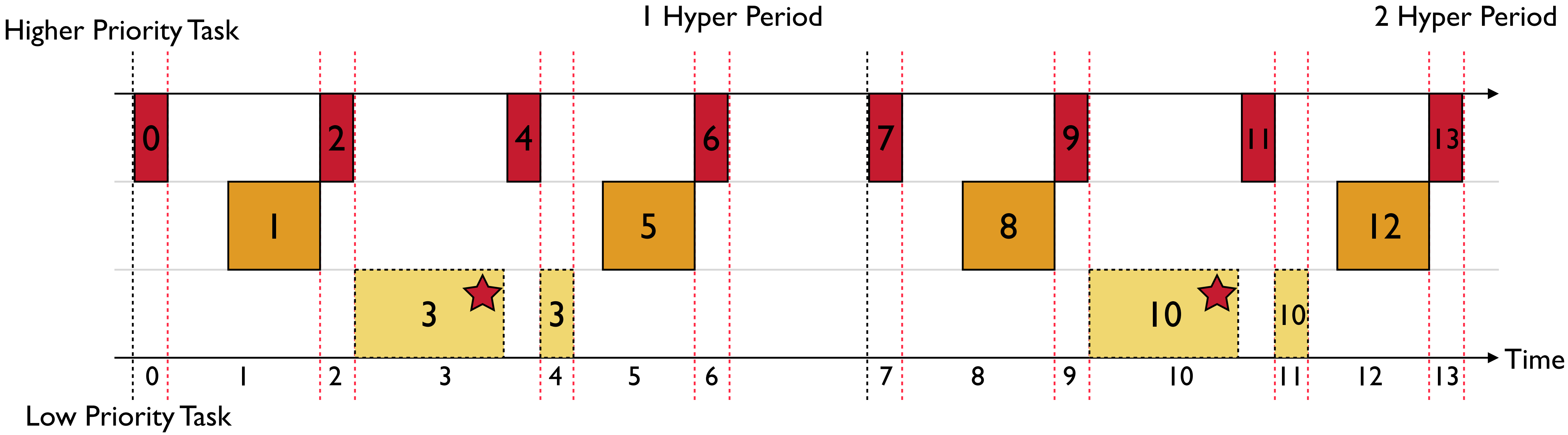

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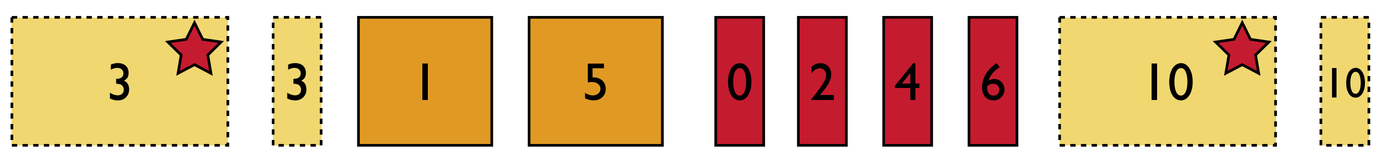

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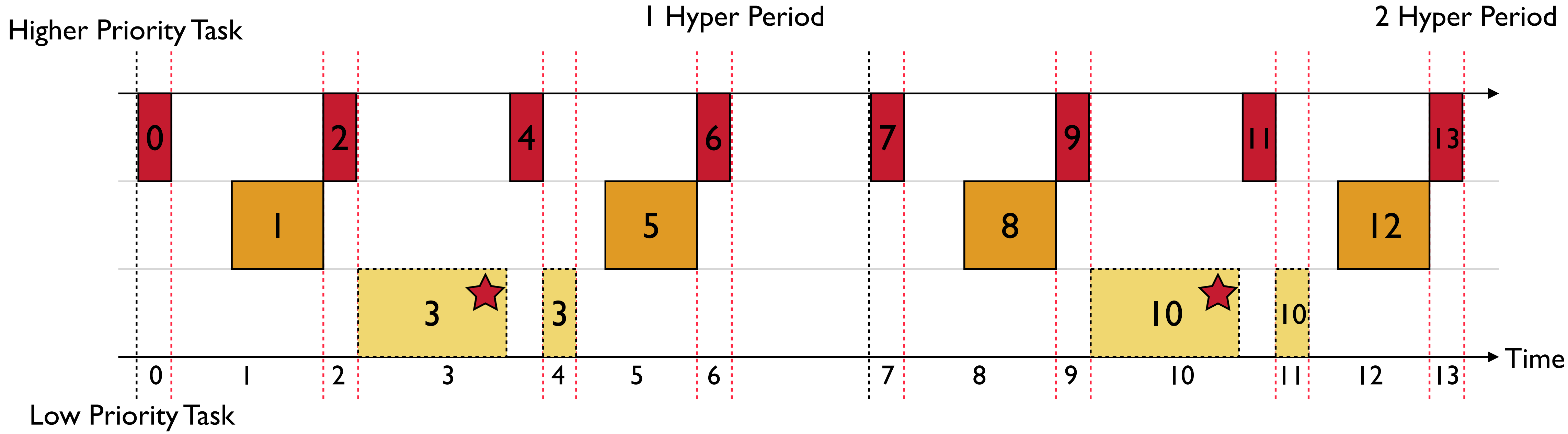

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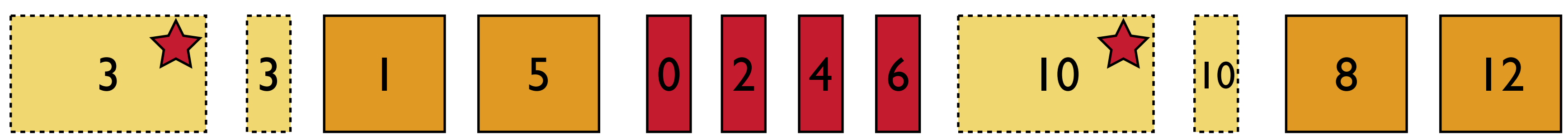

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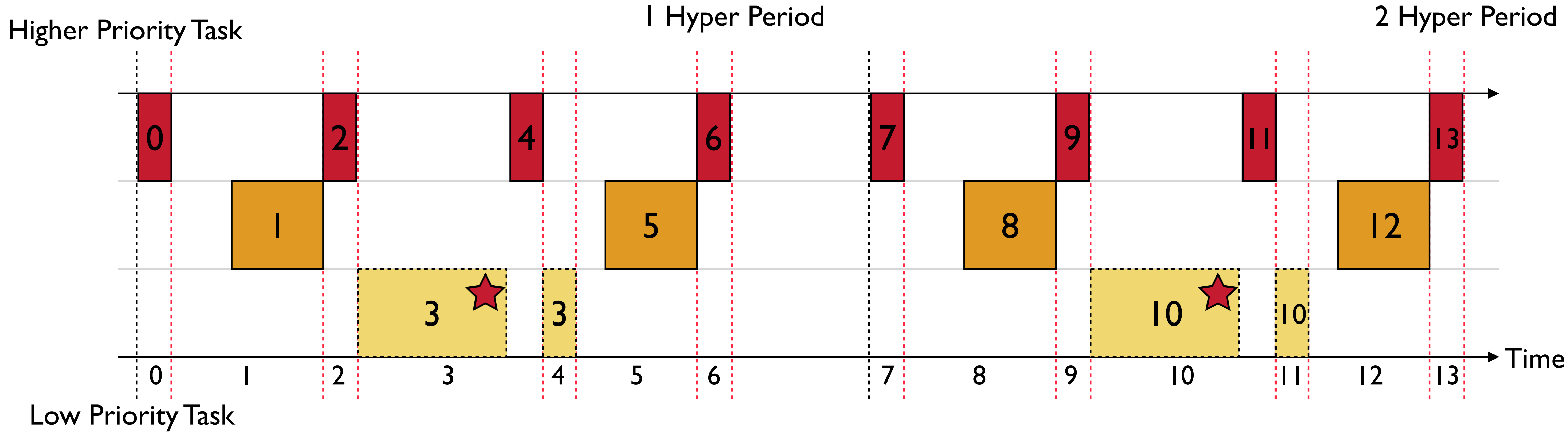

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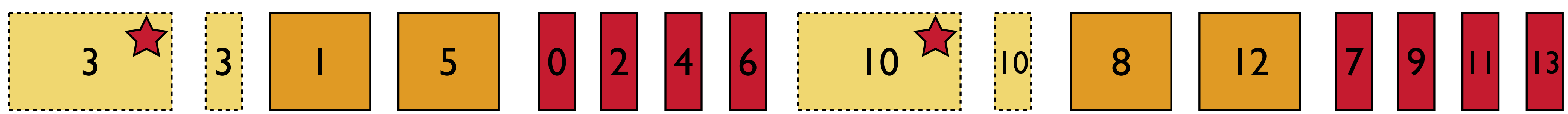
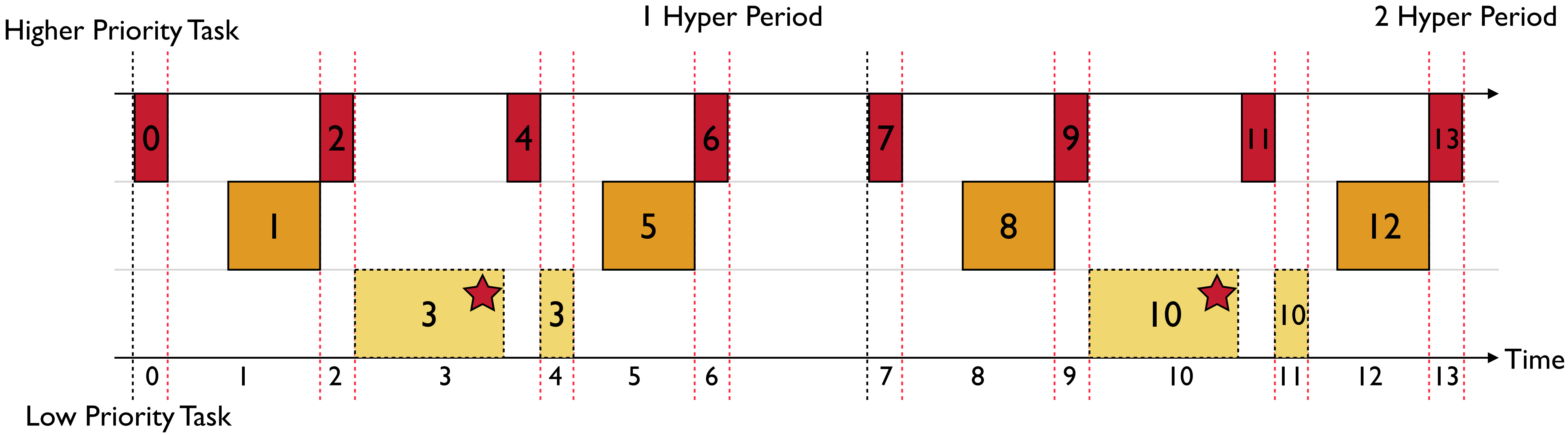


Observations



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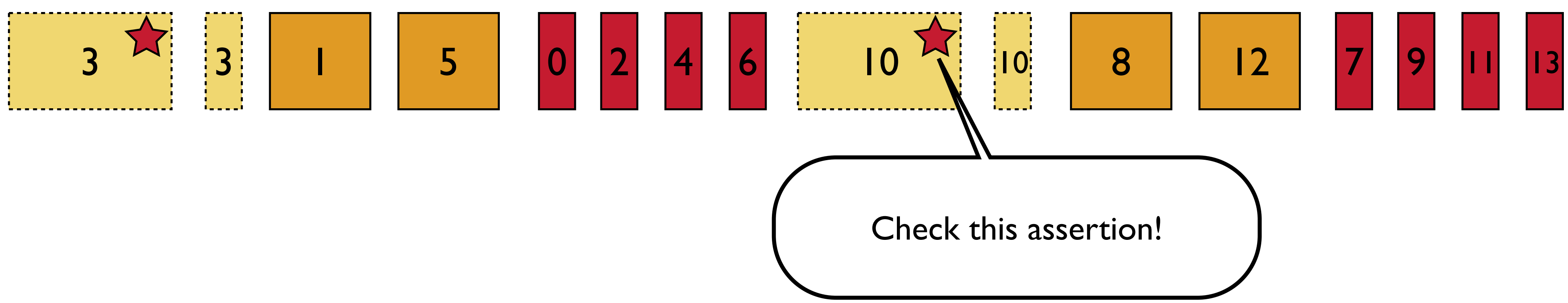

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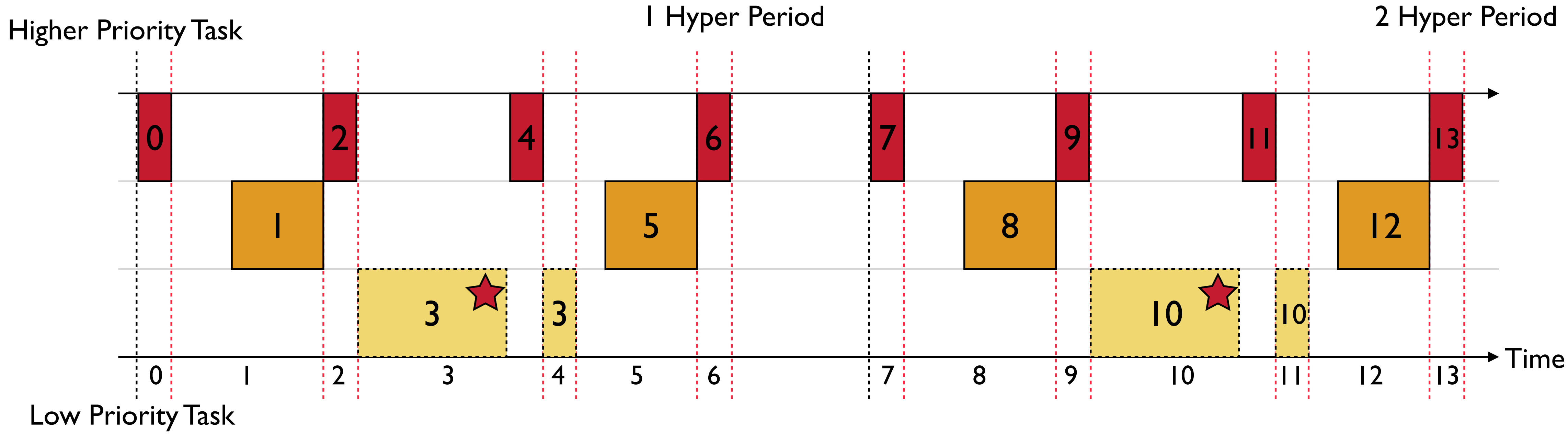

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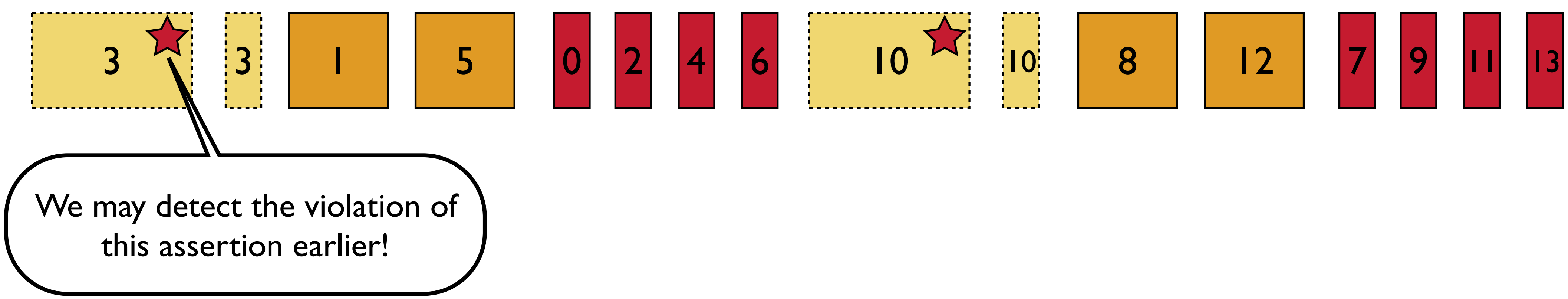
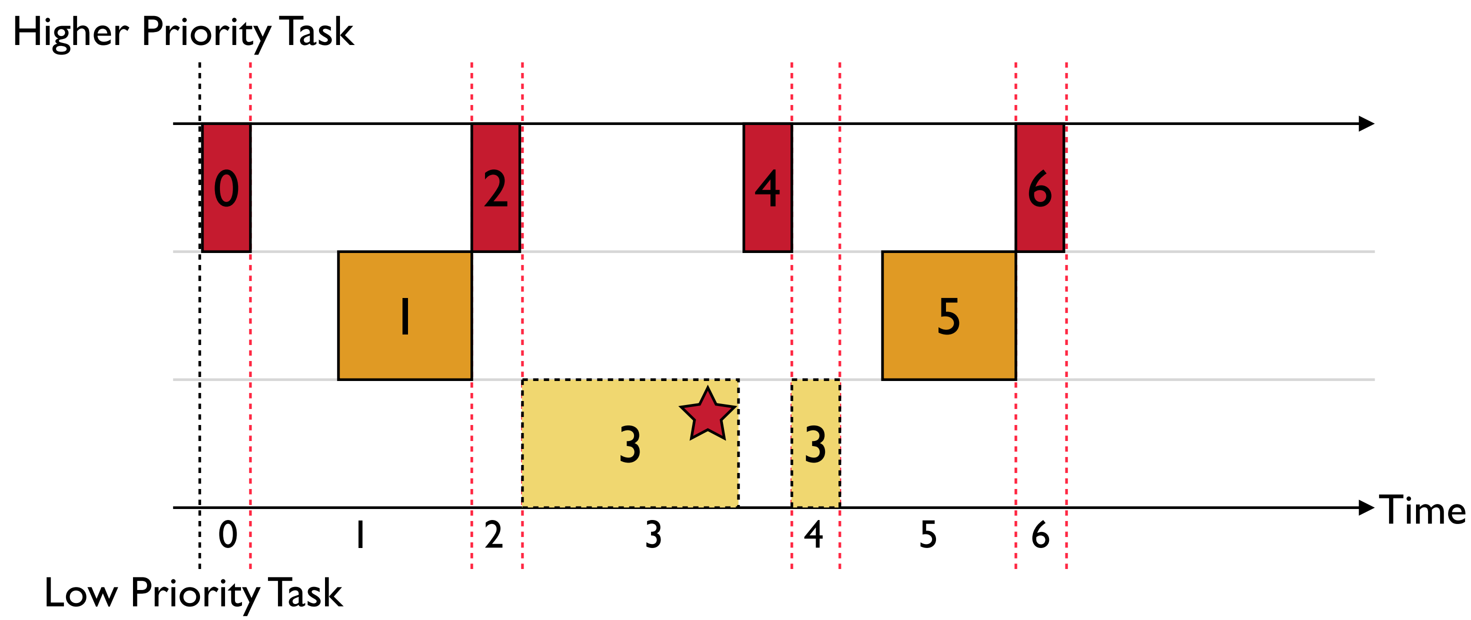
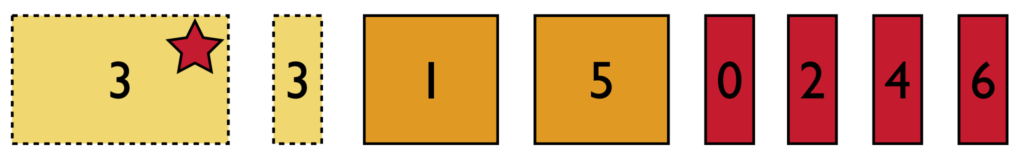


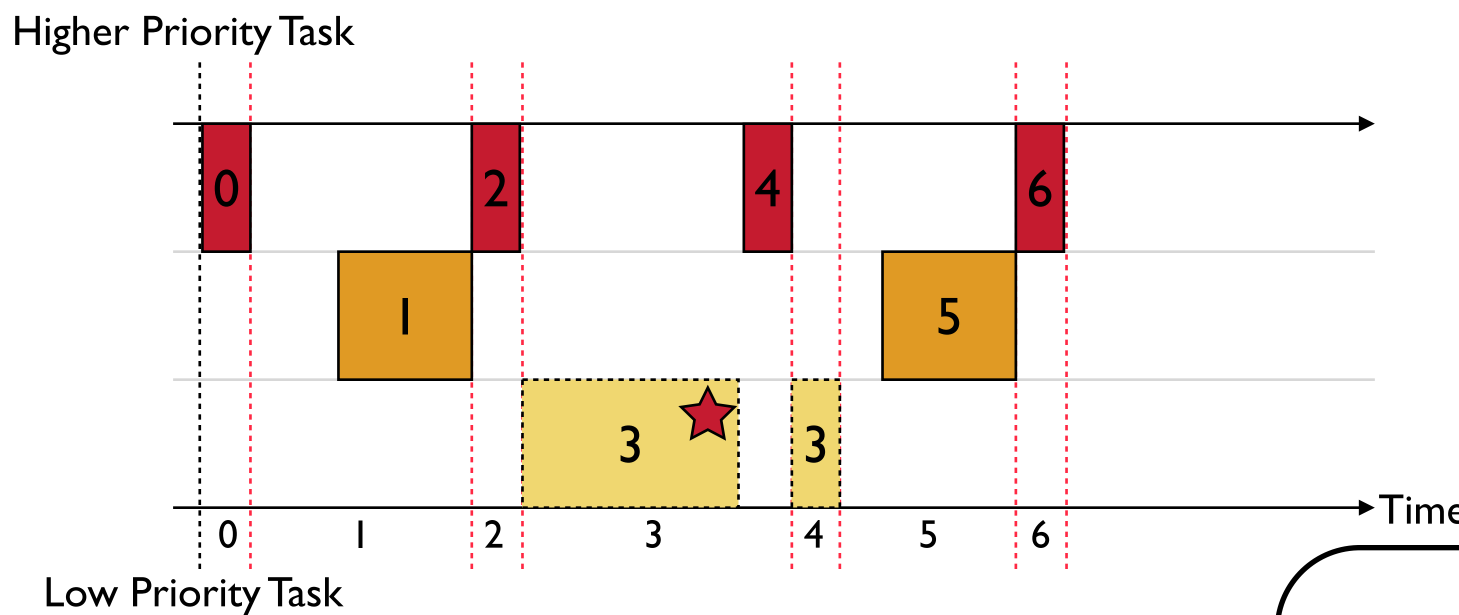
Observations



We executed jobs in the order of their priorities.

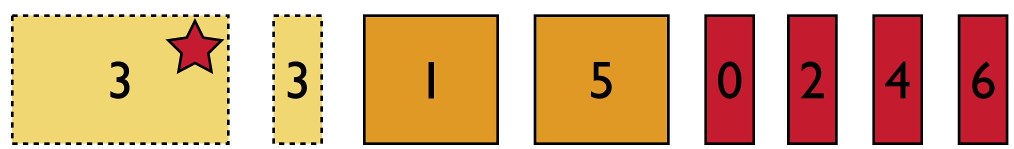


Observations

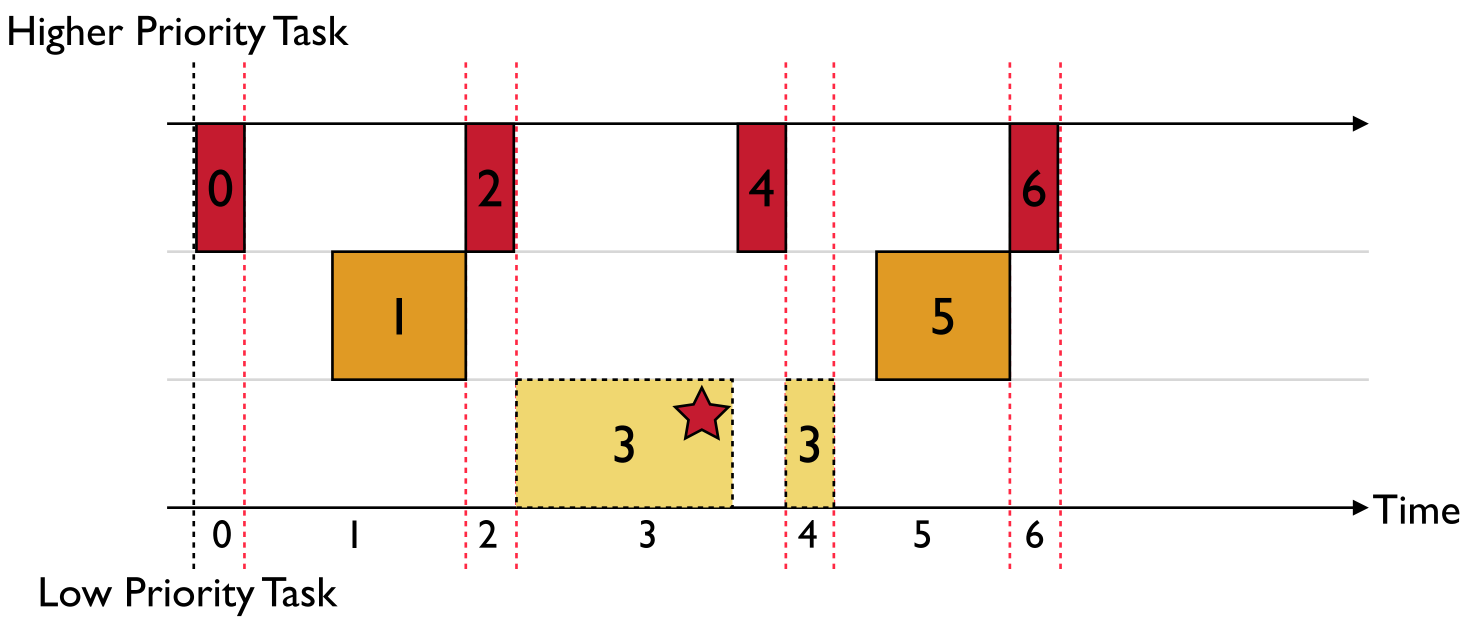


Can we do better?

We executed jobs in the order of their priorities.



Observations



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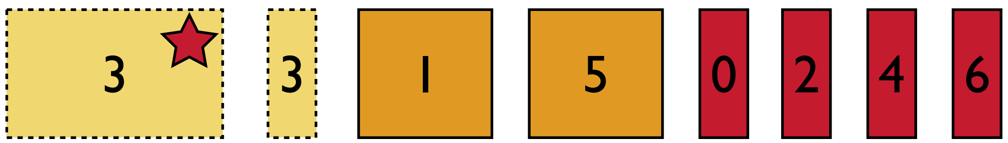
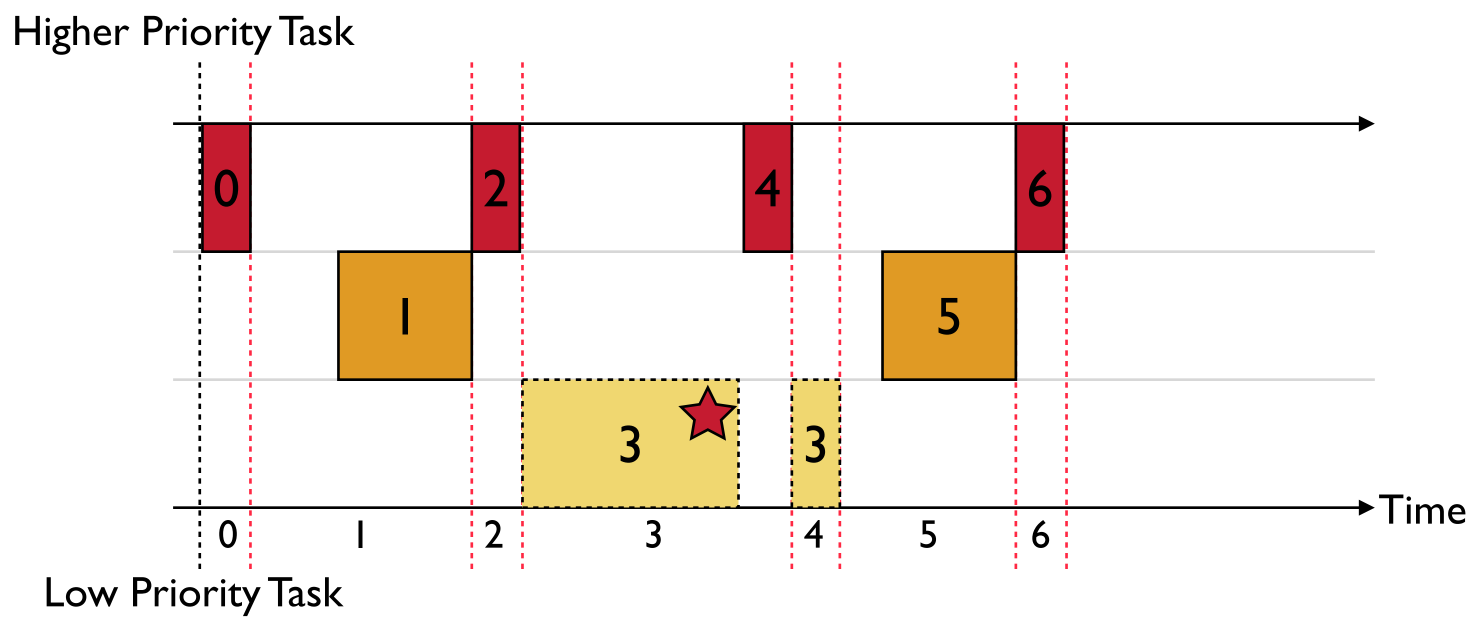


Order jobs by $\sqsubset$ relation

$$j_1 \sqsubset j_2$$

Observations



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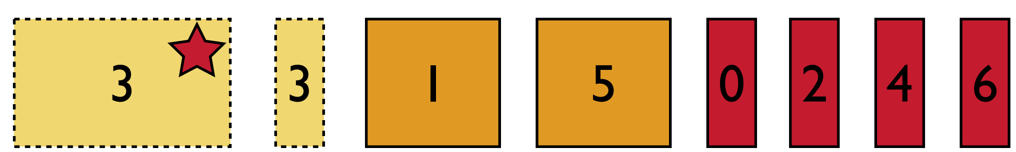
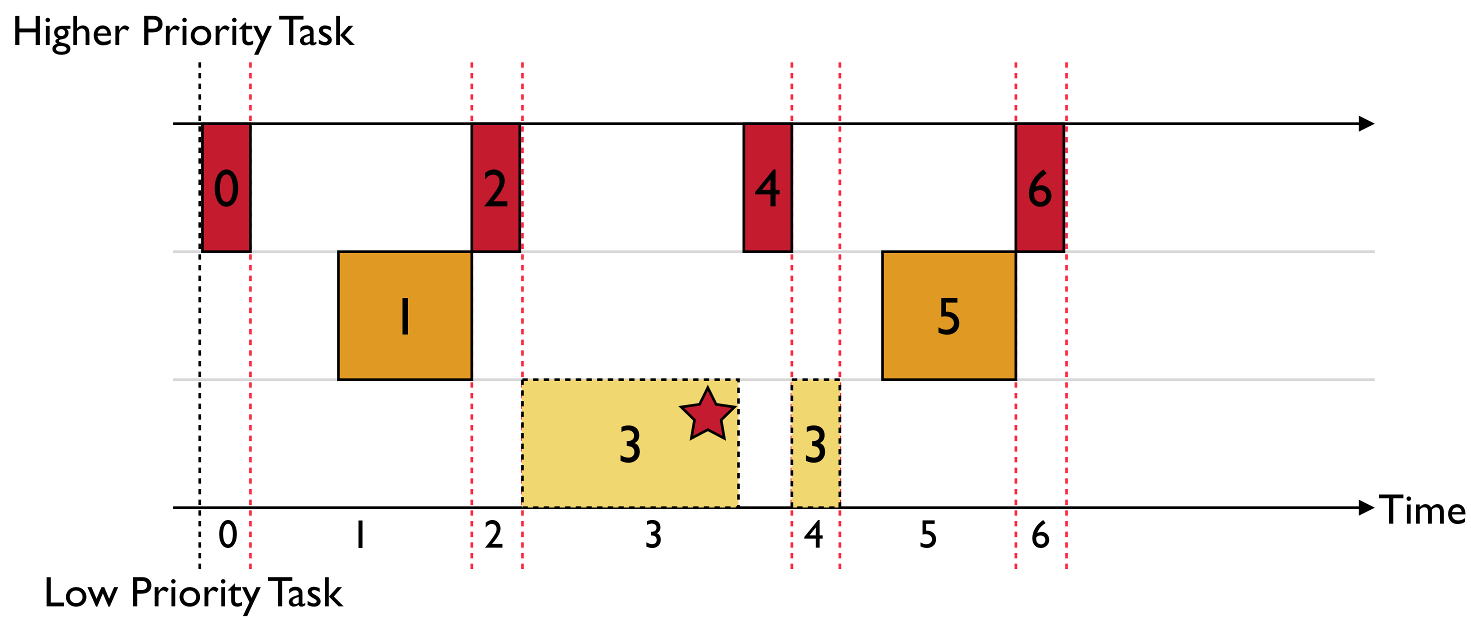


Order jobs by $\sqsubset$ relation

$$j_1 \sqsubset j_2 \begin{cases} \text{j1 ends before j2 starts} \\ \\ \end{cases}$$

Observations



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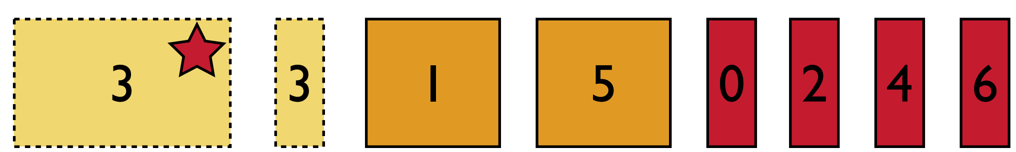
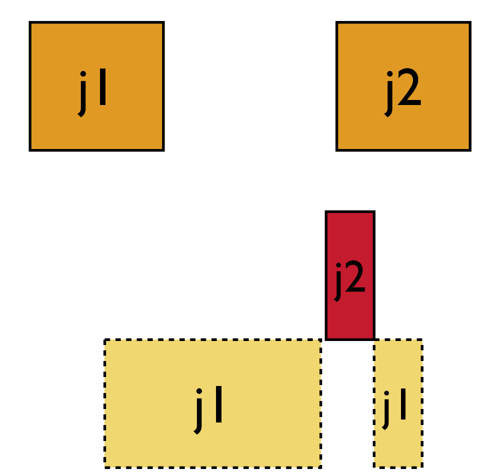


Order jobs by $\sqsubset$ relation

$$j_1 \sqsubset j_2 \begin{cases} \text{j1 ends before j2 starts} \\ \text{or} \\ \text{j1 can be preempted by j2} \end{cases}$$



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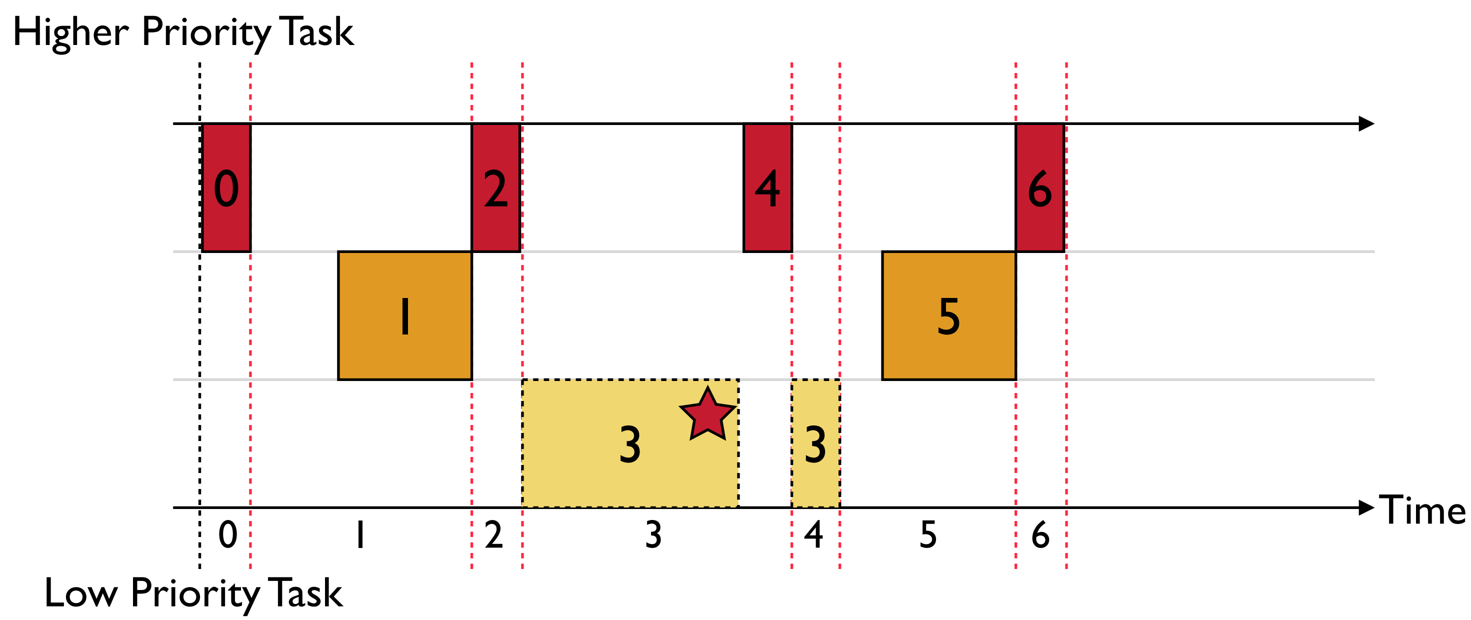

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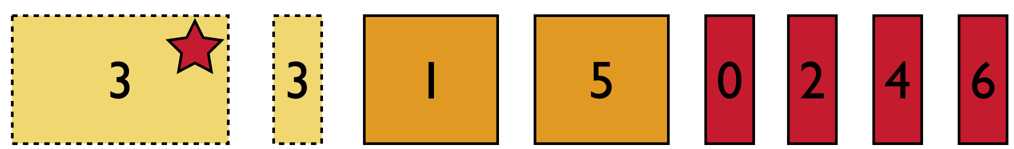

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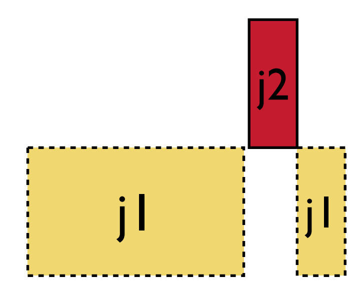

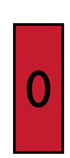

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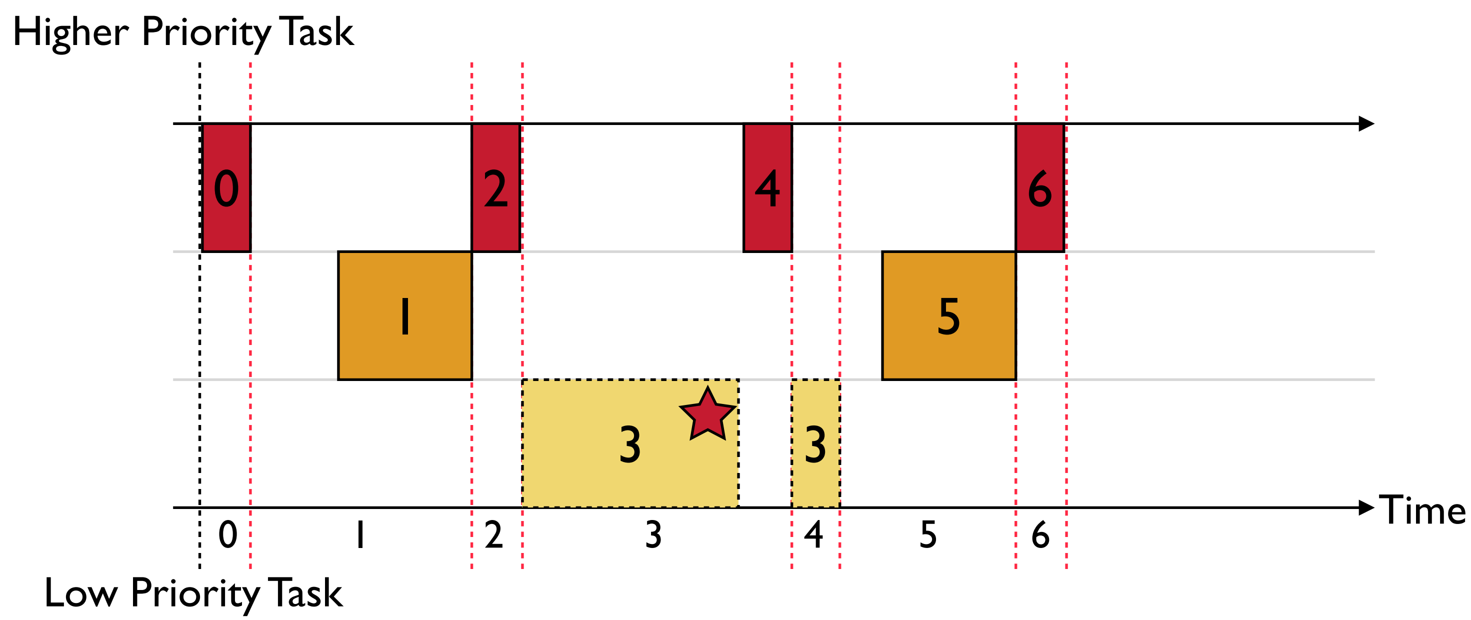

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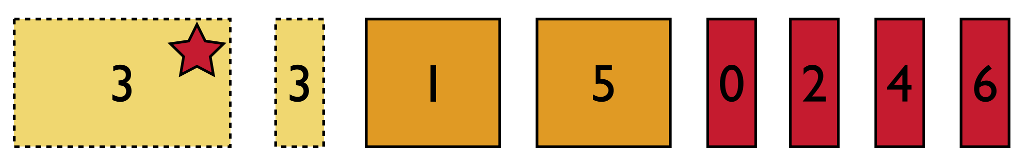

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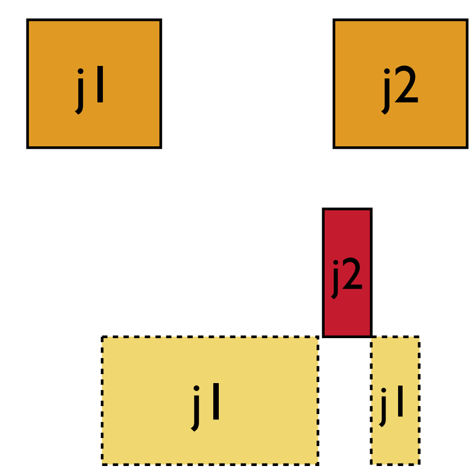
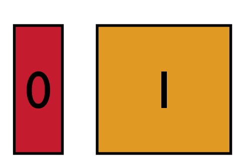
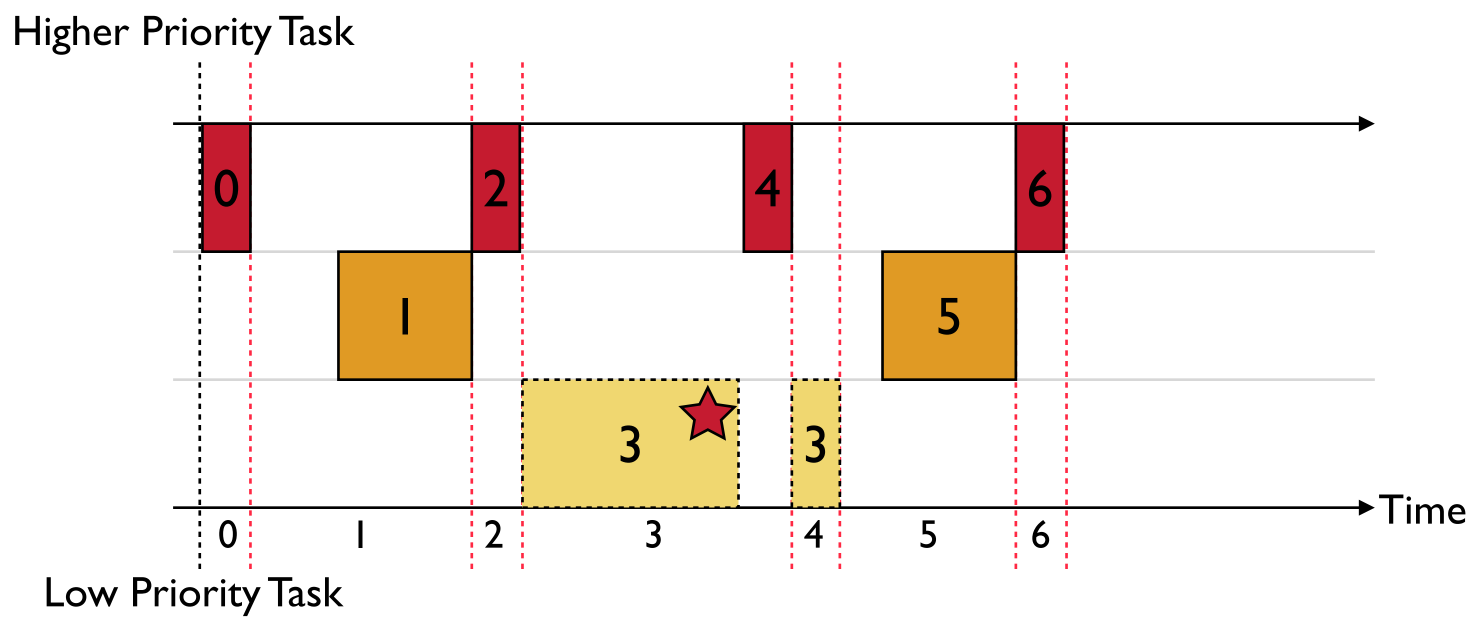




Observations



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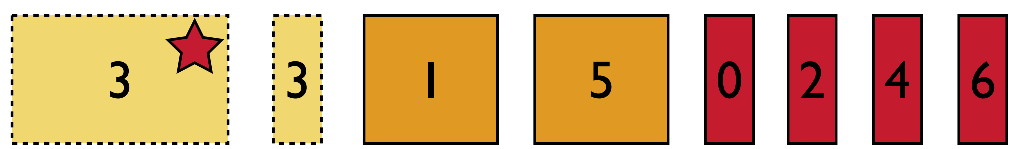

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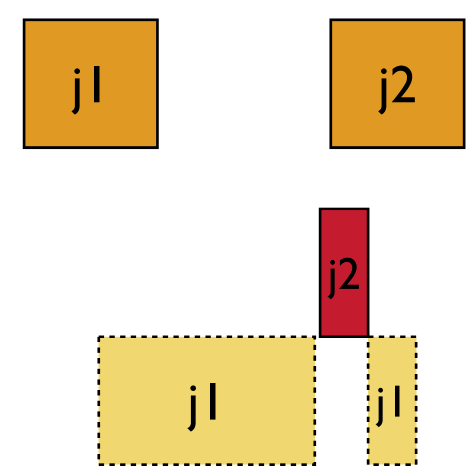

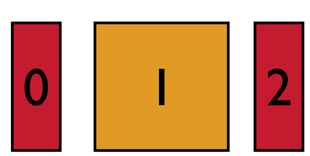

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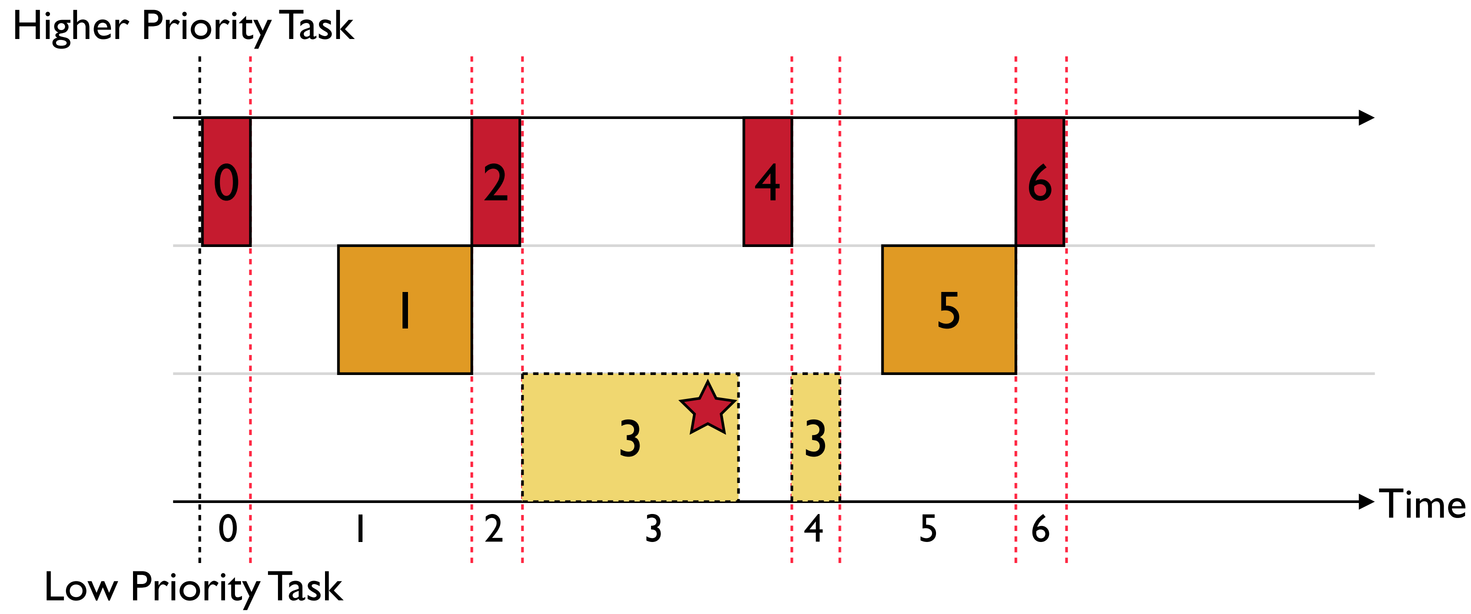

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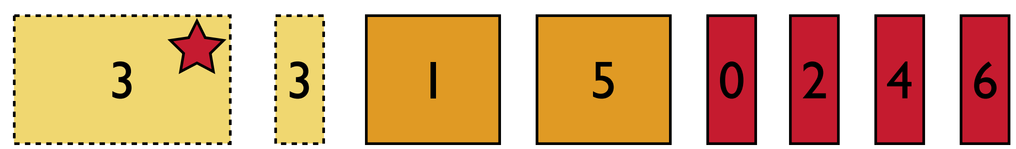

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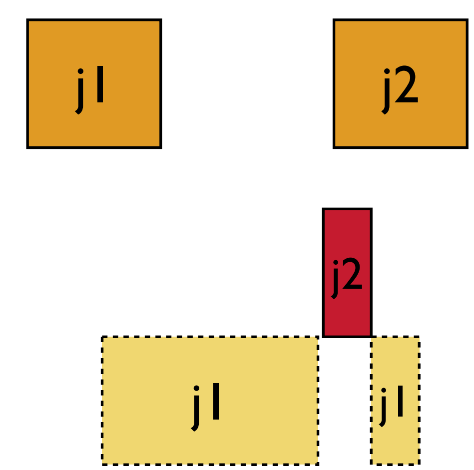
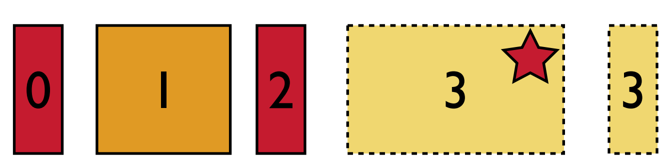
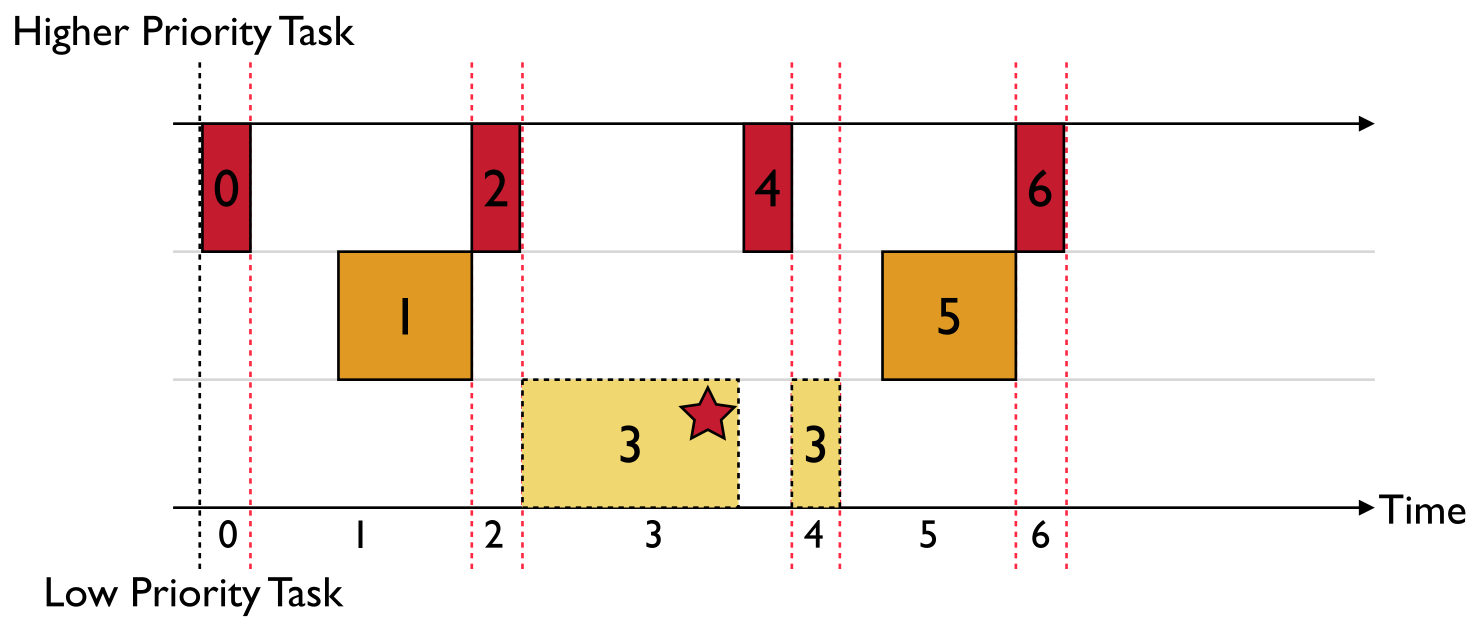
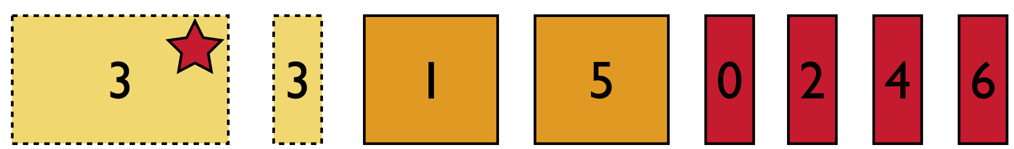




Observations

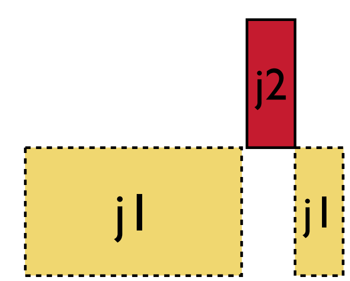


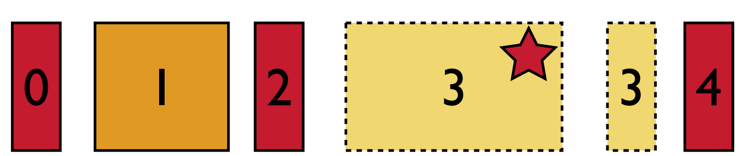
We executed jobs in the order of their priorities.



Order jobs by $\sqsubset$ relation

$$j_1 \sqsubset j_2 \begin{cases} \text{j1 ends before j2 starts} \\ \text{or} \\ \text{j1 can be preempted by j2} \end{cases}$$





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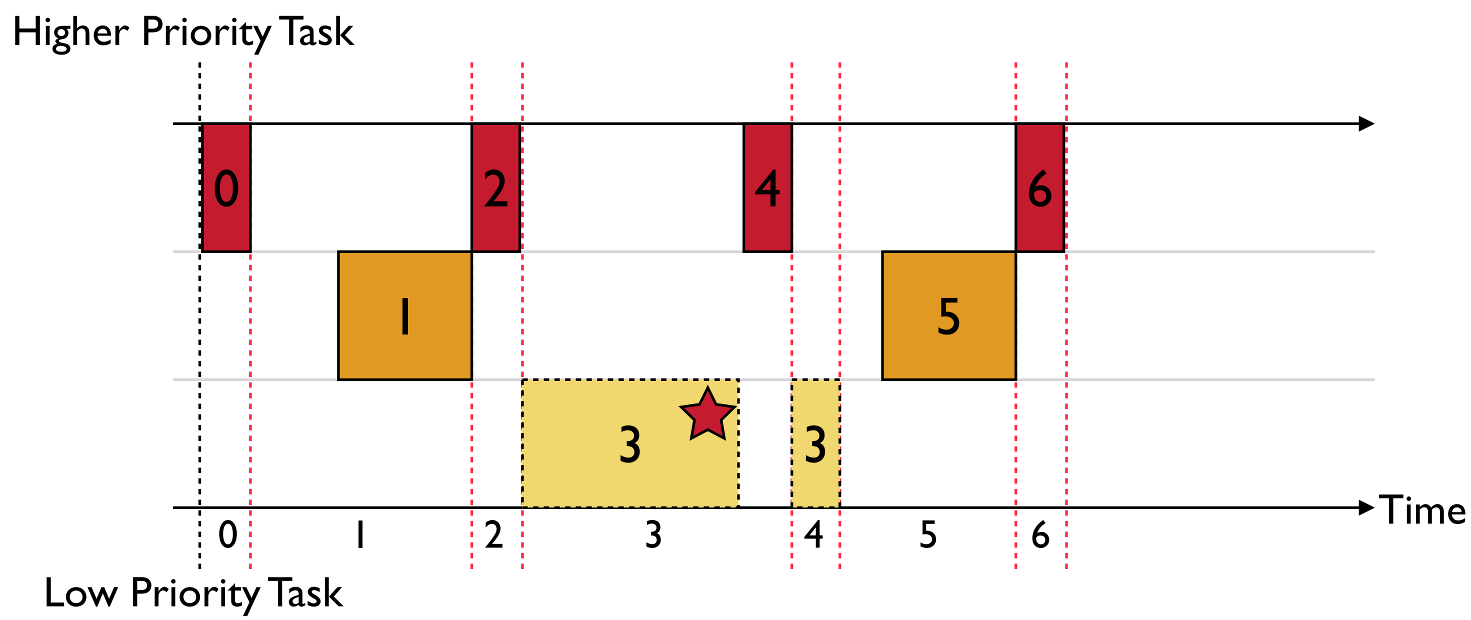

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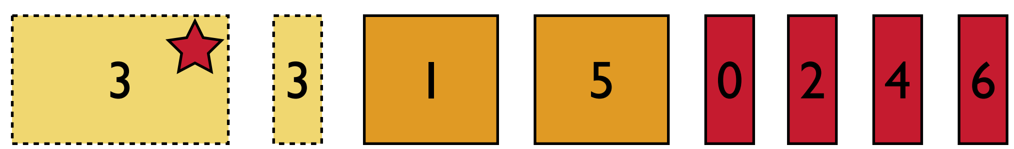

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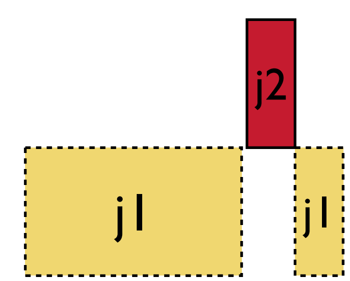

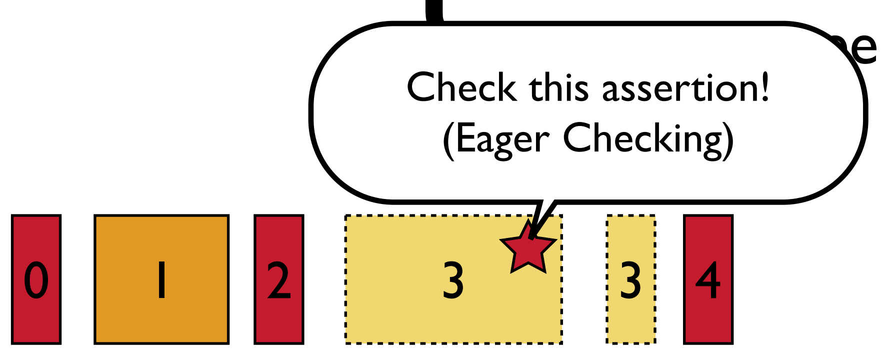

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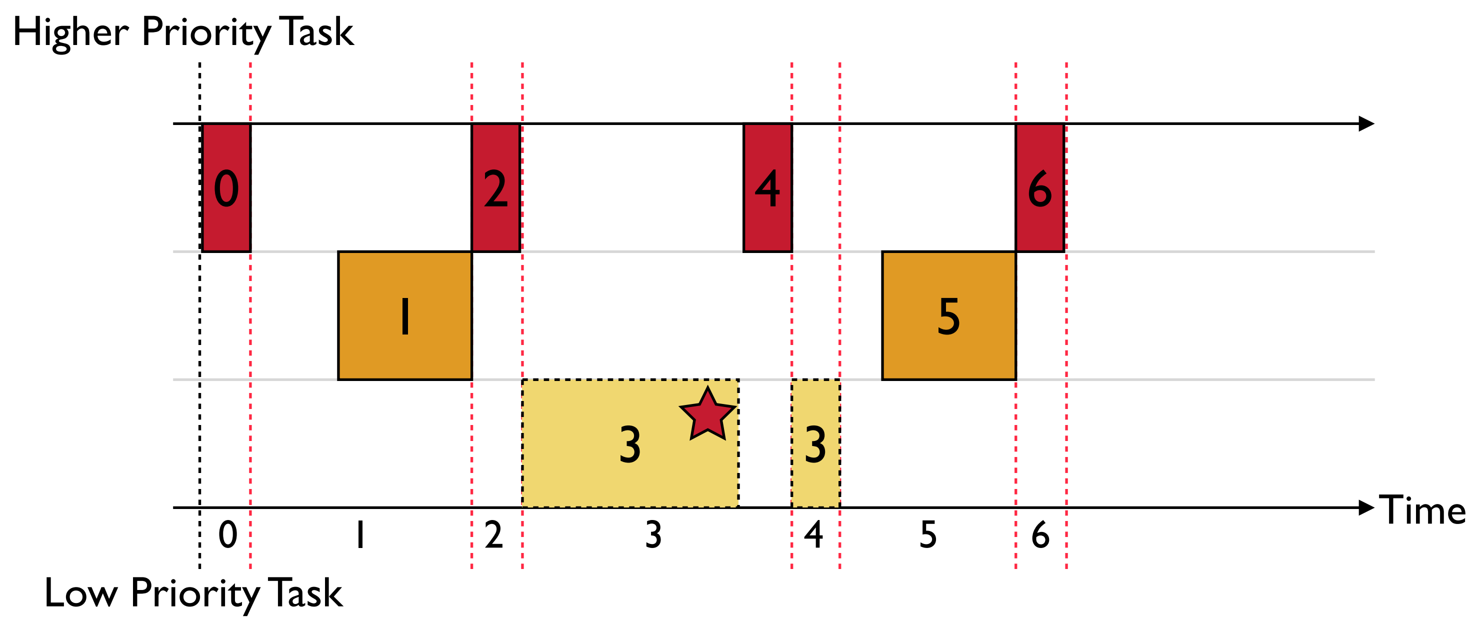

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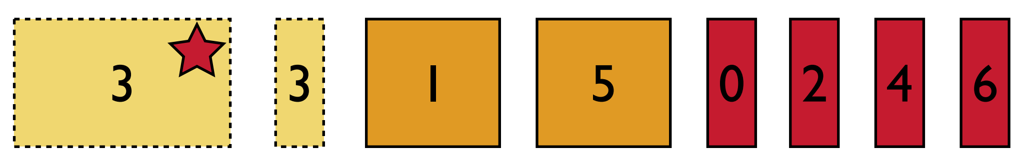

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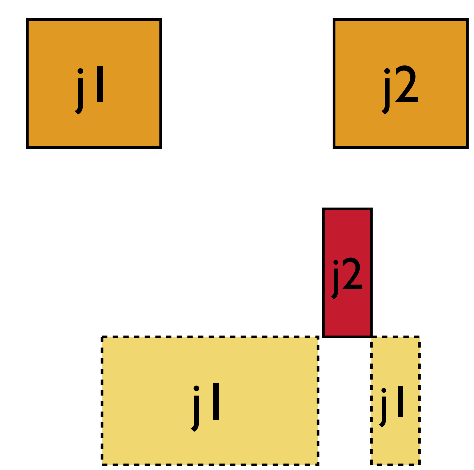

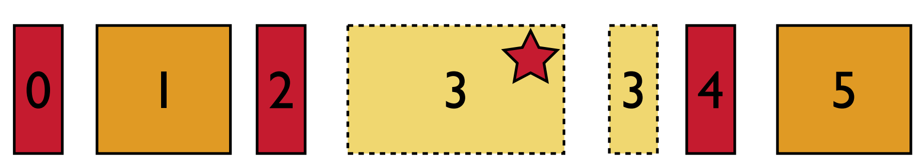

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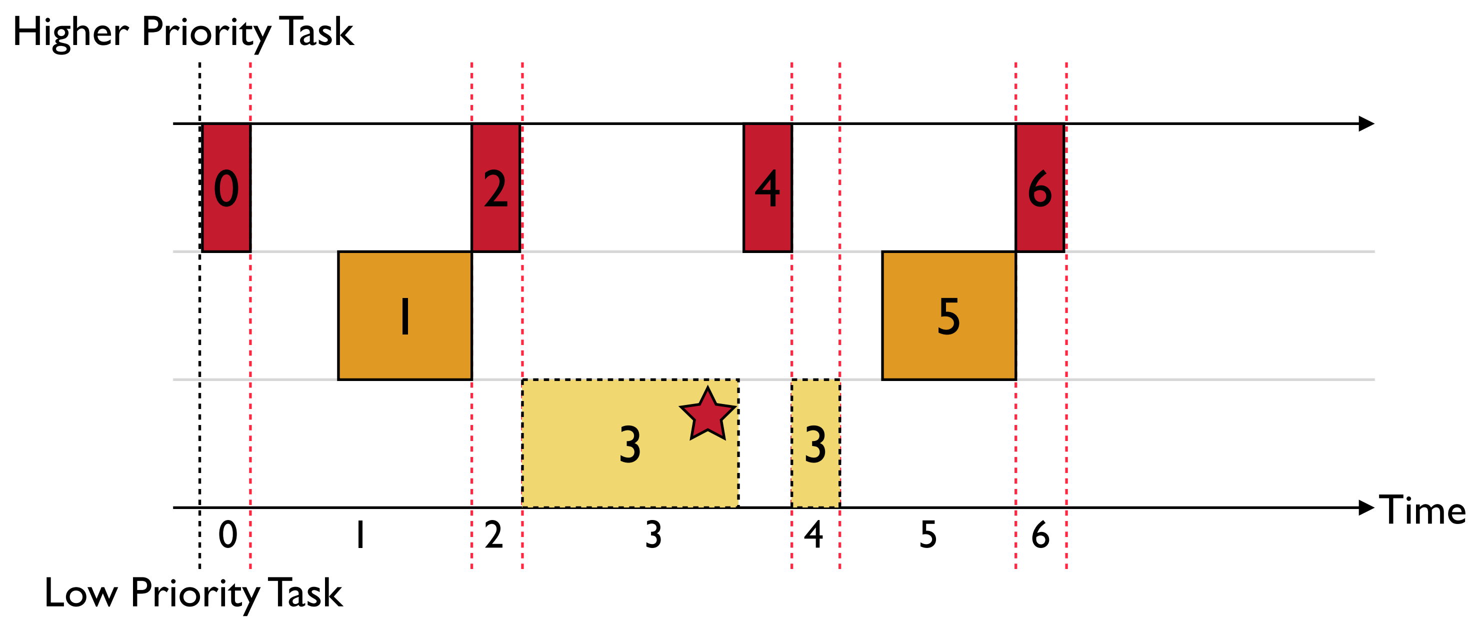

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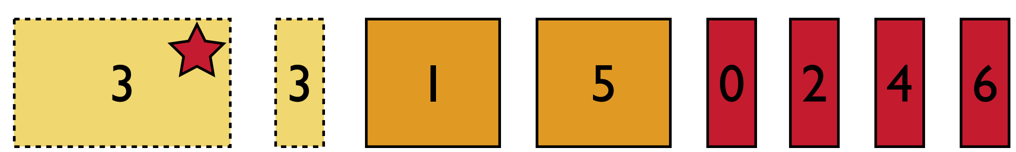

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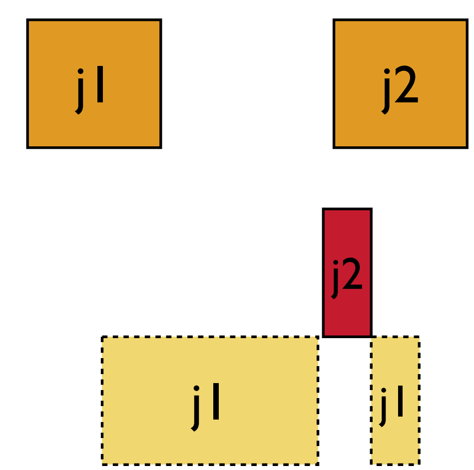

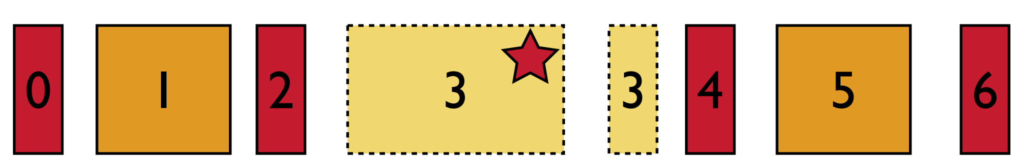

Algorithm 1 The sequentialization $\mathcal{S}$ of the time-bounded periodic program $\mathcal{C}_{\mathcal{H}}$. Notation: J is the set of all jobs; G is the set of global variables of $\mathcal{C}$; i_g is the initial value of g according to $\mathcal{C}$; '$*$' is a non-deterministic value.

var $rnd, start[\,], end[\,], localAssert[\,]$
$\forall g \in \mathsf{G} \centerdot$ **var** $g[\,], v_g[\,]$

function MAIN( )
    $\forall g \in \mathsf{G} \centerdot\ g[0] := i_g$
    HYPERPERIOD()

function HYPERPERIOD( )
    SCHEDULEJOBS()
    $\forall g \in \mathsf{G} \centerdot\ \forall r \in [1, R) \centerdot$
        $v_g[r] := *; g[r] := v_g[r]$
    let the ordering of jobs by $\sqsubset$ *be*
    $j_0 \sqsubset j_1 \sqsubset \ldots j_{R-1}$
    RUNJOB(j_0); ...; RUNJOB(j_{R-1})

function SCHEDULEJOBS( )
    $\forall j \in \mathsf{J} \centerdot\ start[j] = *; end[j] = *$
    // *Jobs are sequential*
    $\forall i \in [0, N) \centerdot\ \forall k \in [0, J_i) \centerdot$ assume
        $(0 \le start[\mathsf{J}(i,k)] \le end[\mathsf{J}(i,k)] < R)$
    // *Jobs are well-separated*
    $\forall j_1 \lhd j_2 \centerdot$ assume$(end[j_1] < start[j_2])$
    $\forall j_1 \uparrow j_2 \centerdot$ assume$(start[j_1] \le start[j_2])$
    // *Jobs are well-nested*
    $\forall j_1 \uparrow j_2 \centerdot$ assume$(start[j_2] \le end[j_1]$
        $\implies (start[j_2] \le end[j_2] < end[j_1]))$

function RUNJOB(Job j)
    $localAssert[j] := 1$
    $rnd := start[j]$
    $\hat{T}(j)$
    assume$(rnd = end[j])$
    if $rnd < R - 1$ **then**
        $\forall g \in \mathsf{G} \centerdot$ assume
            $(g[rnd] = v_g[rnd + 1])$
    $X := \{j' \mid (j' = j \lor j' \uparrow j) \land$
        $(\forall j'' \neq j \centerdot\ j' \uparrow j'' \Rightarrow j'' \sqsubset j)\}$
    $\forall j' \in X \centerdot$ assert$(localAssert[j'])$

function $\hat{T}$(Job j)
    Obtained from T_t *by replacing*
    each statement 'st' with:
    CS(j) ; $st[g \leftarrow g[rnd]]$
    and each 'assert(e)' with:
    $localAssert[j] := e$

function CS(Job j)
    if $(*)$ **then return** FALSE
    $o := rnd\,; rnd := *$
    assume$(o < rnd \le end[j])$
    $\forall j' \in \mathsf{J} \centerdot\ j \uparrow j' \implies$
        assume$(rnd \le start[j'] \lor$
            $rnd > end[j'])$
    return TRUE

CompSeq

Algorithm 1 The sequentialization $\mathcal{S}$ of the time-bounded periodic program $\mathcal{C}_{\mathcal{H}}$. Notation: J is the set of all jobs; G is the set of global variables of $\mathcal{C}$; i_g is the initial value of g according to $\mathcal{C}$; '$*$' is a non-deterministic value.

```
var rnd, start[], end[], localAssert[]
∀g ∈ G . var g[], v_g[]

function MAIN( )
    ∀g ∈ G . g[0] := i_g
    HYPERPERIOD()

function HYPERPERIOD( )
    SCHEDULEJOBS()
    ∀g ∈ G . ∀r ∈ [1, R).
        v_g[r] := ∗; g[r] := v_g[r]
    let the ordering of jobs by ⊏ be
    j_0 ⊏ j_1 ⊏ ... j_{R−1}
    RUNJOB(j_0); ... ; RUNJOB(j_{R−1})

function SCHEDULEJOBS( )
    ∀j ∈ J . start[j] = ∗; end[j] = ∗
  // Jobs are sequential
    ∀i ∈ [0, N) . ∀k ∈ [0, J_i) . assume
      (0 ≤ start[J(i, k)] ≤ end[J(i, k)] < R)
  // Jobs are well-separated
    ∀j_1 ◁ j_2 . assume(end[j_1] < start[j_2])
    ∀j_1 ↑ j_2 . assume(start[j_1] ≤ start[j_2])
  // Jobs are well-nested
    ∀j_1 ↑ j_2 . assume(start[j_2] ≤ end[j_1]
      ⟹ (start[j_2] ≤ end[j_2] < end[j_1]))

function RUNJOB(Job j)
    localAssert[j] := 1
    rnd := start[j]
    T̂(j)
    assume(rnd = end[j])
    if rnd < R − 1 then
        ∀g ∈ G . assume
            (g[rnd] = v_g[rnd + 1])
    X := {j′ | (j′ = j ∨ j′ ↑ j) ∧
        (∀j″ ≠ j . j′ ↑ j″ ⇒ j″ ⊏ j)}
    ∀j′ ∈ X . assert(localAssert[j′])

function T̂(Job j)
Obtained from T_t by replacing
  each statement 'st' with:
    CS(j) ; st[g ← g[rnd]]
  and each 'assert(e)' with:
    localAssert[j] := e

function CS(Job j)
    if (∗) then return FALSE
    o := rnd ; rnd := ∗
    assume(o < rnd ≤ end[j])
    ∀j′ ∈ J . j ↑ j′ ⟹
        assume(rnd ≤ start[j′]∨
            rnd > end[j′])
    return TRUE
```

CompSeq

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    $\forall j_1 \uparrow j_2\,\centerdot$ assume$(start[j_2] \le end[j_1]$
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        $\forall g \in \mathsf{G}\,\centerdot$ assume
            $(g[rnd] = v_g[rnd + 1])$
    $X := \{j' \mid (j' = j \vee j' \uparrow j) \wedge$
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      $rnd > end[j'])$
  return TRUE

Naive Approach:

1. Enumerate all possible (sequentialized) executions
2. Verify each of them

Exponential Blow-up!

MonoSeq/CompSeq:

1. Construct a **non-deterministic** sequentialized program
2. Enforce legal job scheduling and prune out infeasible thread executions by adding **constraints**

$O(R^2)$
where R= # of Jobs

Sequentialization Algorithms

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HarmonicSeq: Only for **Harmonic** Periodic Program

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1. Construct a **non**[...]tialized program
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$P_i \le P_j \implies P_i | P_j$

Sequentialization Algorithms

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1. Construct a **non**...
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Common in Real-time Embedded Systems:

1. 100% CPU Utilization
2. More predictable battery usage

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1. Construct a **non-deterministic** sequentialized program
2. Enforce legal job scheduling and prune out infeasible thread executions by adding **constraints**

$O(R \cdot N)$
where N= # of tasks, usually exponentially smaller than R

Sequentialization Algorithms

Algorithm 2 Procedure to assign legal starting and ending rounds to jobs in a harmonic program.

var $min[\,], max[\,]$ *//extra variables*

function scheduleHarmonic( )

$\quad \forall j \in \mathsf{J} \,.\; start[j] = *; end[j] = *; min[j] = *; max[j] = *$

// Correctness of min and max

$\quad \forall n \in \mathcal{T} \,.\; isleaf(n) \implies \text{assume}(min[n] = start[n] \wedge max[n] = end[n])$

$\quad \forall n \in \mathcal{T} \,.\; \neg isleaf(n) \implies \text{assume}(min[n] = \text{MIN}(start[n], min[first(n)]))$

$\quad \forall n \in \mathcal{T} \,.\; \neg isleaf(n) \implies \text{assume}(max[n] = \text{MAX}(end[n], max[last(n)]))$

// Jobs are sequential

$\quad \forall n \in \mathcal{T} \,.\; \text{assume}(low(n) \leq start[n] \leq end[n] \leq high(n))$

// Jobs are well-separated

$\quad \forall n \in \mathcal{T} \,.\; hasNext(n) \implies \text{assume}(max[n] < min[next(n)])$

$\quad \forall j_1 \uparrow j_2 \,.\; \text{assume}(start[j_1] \leq start[j_2])$

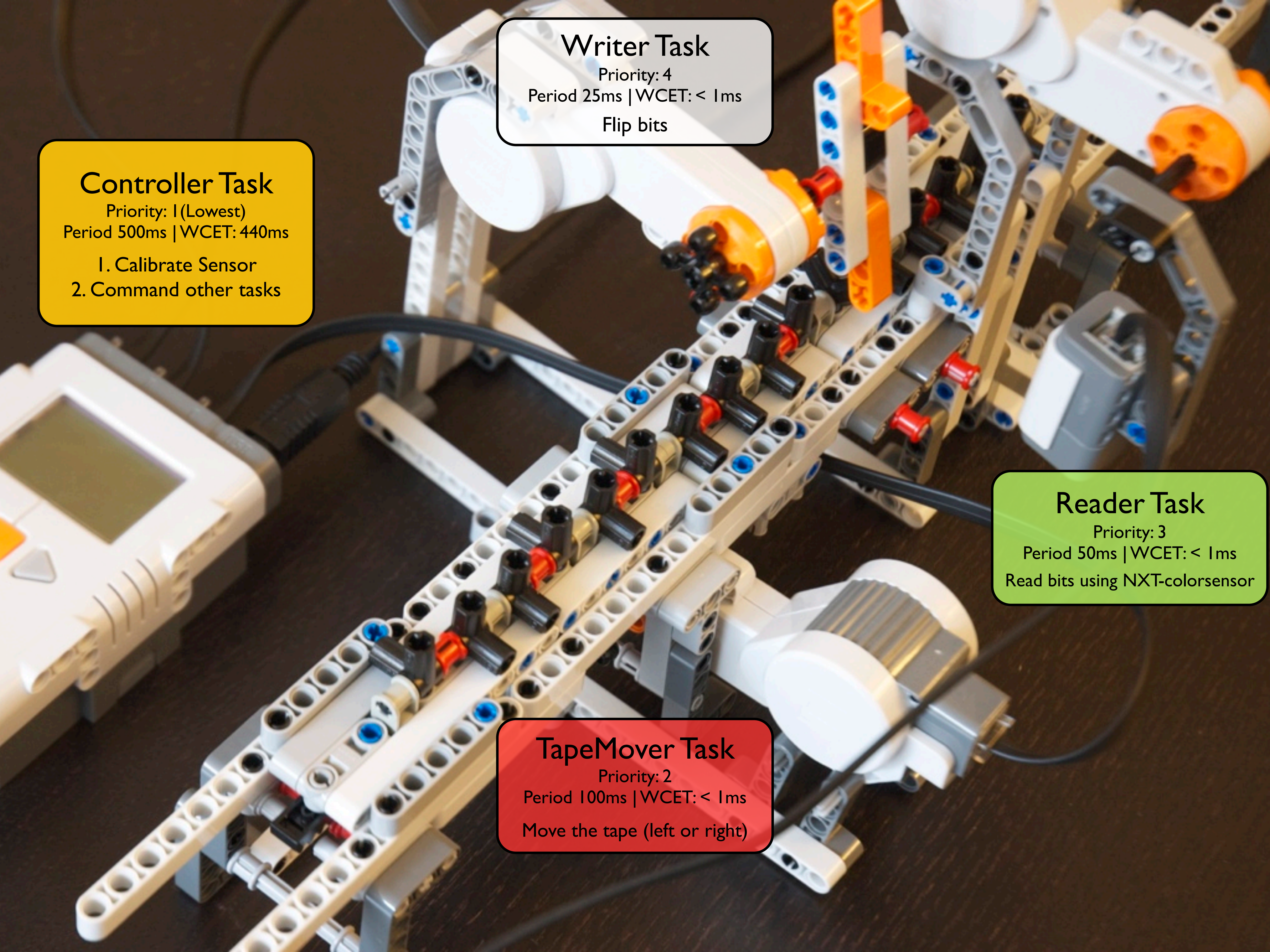
// Jobs are well-nested

$\quad \forall j_1 \uparrow j_2 \,.\; \text{assume}(start[j_2] \leq end[j_1] \implies (start[j_2] \leq end[j_2] < end[j_1]))$

$\mathcal{T}(n)$	= sub-tree of $\mathcal{T}$ rooted at n	$isleaf(n)$	= true iff n is a leaf node
$level(n)$	= level of node n	$size(n)$	= number of nodes in $\mathcal{T}(n)$
$id(n)$	= position of n in the DFS order of $\mathcal{T}$	$hasNext(n)$	= true iff n is *not* the last node at level $level(n)$
$next(n)$	= node after n at level $level(n)$	$first(n)$	= first child of n
$last(n)$	= last child of n	$maxid(n)$	$= id(n) + size(n) - 1$
$low(n)$	$= id(n) - level(n)$	$high(n)$	$= maxid(n)$

Case Study:

Concurrent Turing Machine



Writer Task
Priority: 4
Period 25ms | WCET: < 1ms
Flip bits
Controller Task
Priority: 1 (Lowest)
Period 500ms | WCET: 440ms
1. Calibrate Sensor
2. Command other tasks
Reader Task
Priority: 3
Period 50ms | WCET: < 1ms
Read bits using NXT-colorsensor
TapeMover Task
Priority: 2
Period 100ms | WCET: < 1ms
Move the tape (left or right)

Properties

Property 2: When writer flips a bit, the tape motor and read motor should **stop**.

Controller Task

```
case C_WRITE:

  /* Check if we need to chagne the bit */
  if(R(input) != R(output)) {
    /* Check the header and move it back if necessary */
    if(nxt_motor_get_count(READ_MOTOR) > 0 && R(R_state) == READ_IDLE) {
      W(R_state, READ_MOVE_HEADER_BACKWARD);
    }

    /* Check the header and flip the bit if it is safe to do */
    if(nxt_motor_get_count(READ_MOTOR) <= 0 && R(W_state) == WRITE_IDLE) {
      W(W_state, WRITE_FLIP);
    }
  } else {
    /* Nothing to change for writer */
    W(W_state, WRITE_IDLE);
    C_state = C_MOVE;
  }
  break;
```

Writer Task

```
  case WRITE_FLIP:
#ifdef VERIFICATION
    /* Property 3: When writer flips a bit, the tape motor and read
       motor should be stopped. */

    /* FAILED!! with BOUND 120 */
    assert(R(T_speed) == 0 && R(R_speed) == 0);
#endif
```

Properties

Property 2: When writer flips a bit, the tap … hould **stop**.

If the READ header is up,
Move it back
to avoid collision!

Controller Task

```
case C_WRITE:

  /* Check if we need to chagne the bi
  if(R(input) != R(output)) {
```

Properties

Property 2: When writer flips a bit, the tap … hould **stop**.

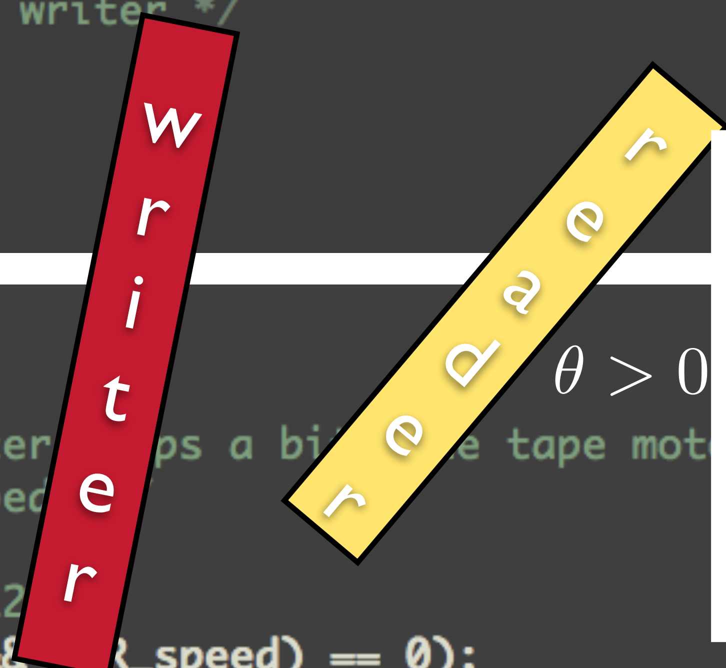
If the READ header is up,
Move it back
to avoid collision!

Controller Task

```
case C_WRITE:

  /* Check if we need to chagne the bi
  if(R(input) != R(output)) {
    /* Check the header and move it back if necessary */
    if(nxt_motor_get_count(READ_MOTOR) > 0 && R(R_state) == READ_IDLE) {
      W(R_state, READ_MOVE_HEADER_BACKWARD);
    }

    /* Check the header and flip the bit if it is safe to do */
    if(nxt_motor_get_count(READ_MOTOR) <= 0 && R(W_state) == WRITE_IDLE) {
      W(W_state, WRITE_FLIP);
    }
  } else {
    /* Nothing to change for write */
    W(W_state, WRITE_IDLE);
    C_state = C_MOVE;
  }
  break;
```

Writer Task

```
  case WRITE_FLIP:
#ifdef VERIFICATION
    /* Property 3: When writer    ps a bi    e tape mot r and read
       motor should be stopped

    /* FAILED!! with BOUND 12
    assert(R(T_speed) == 0 &&     _speed) == 0);
#endif
```



Proper...

Property 2: When writer flips a bit, the tap... ...hould **stop**.

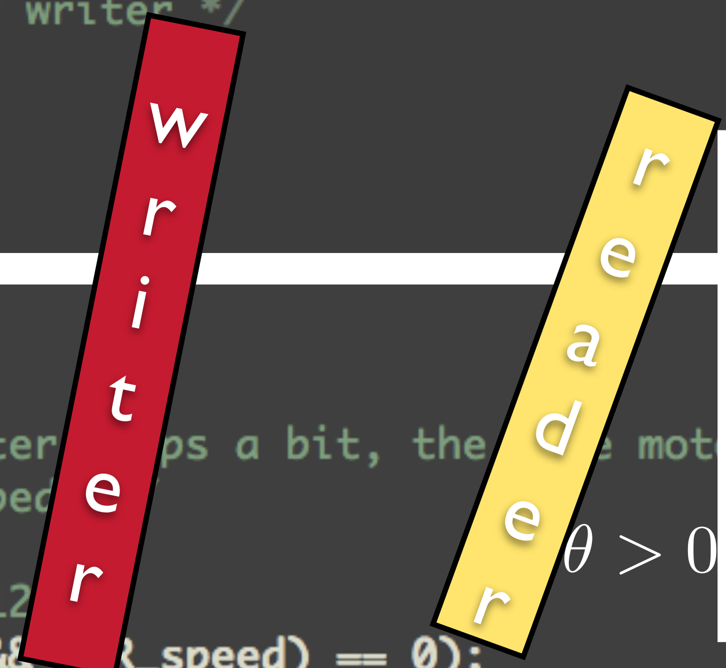
If the READ header is up,
Move it back
to avoid collision!

Controller Task

```
case C_WRITE:

  /* Check if we need to chagne the bi
  if(R(input) != R(output)) {
    /* Check the header and move it back if necessary */
    if(nxt_motor_get_count(READ_MOTOR) > 0 && R(R_state) == READ_IDLE) {
      W(R_state, READ_MOVE_HEADER_BACKWARD);
    }

    /* Check the header and flip the bit if it is safe to do */
    if(nxt_motor_get_count(READ_MOTOR) <= 0 && R(W_state) == WRITE_IDLE) {
      W(W_state, WRITE_FLIP);
    }
  } else {
    /* Nothing to change for write
    W(W_state, WRITE_IDLE);
    C_state = C_MOVE;
  }
  break;
```

Writer Task

```
  case WRITE_FLIP:
#ifdef VERIFICATION
    /* Property 3: When writer   ps a bit, the   e mot r and read
       motor should be stopped

    /* FAILED!! with BOUND 12
    assert(R(T_speed) == 0 &&   _speed) == 0);
#endif
```



Properties

Property 2: When writer flips a bit, the tape motor and read motor should **stop**.

Controller Task

```
case C_WRITE:

  /* Check if we need to chagne the bit
  if(R(input) != R(output)) {
    /* Check the header and move it bac
    if(nxt_motor_get_count(READ_MOTOR)                    ) {
      W(R_state, READ_MOVE_HEADER_BACKW
    }

    /* Check the header and flip the bit if it is safe to do */
    if(nxt_motor_get_count(READ_MOTOR) <= 0 && R(W_state) == WRITE_IDLE) {
      W(W_state, WRITE_FLIP);
    }
  } else {
    /* Nothing to change for writer */
    W(W_state, WRITE_IDLE);
    C_state = C_MOVE;
  }
  break;
```

OK, it's safe to write!

Writer Task

```
  case WRITE_FLIP:
#ifdef VERIFICATION
    /* Property 3: When writer   ps a bit, the tape mot   r   read
       motor should be stopped

    /* FAILED!! with BOUND 12
    assert(R(T_speed) == 0 &&      _speed) == 0);
#endif
```

writer

reader

$\theta <= 0$

Properties

Property 2: When writer flips a bit, the tape motor and read motor should **stop**.

Controller Task

```
case C_WRITE:

  /* Check if we need to chagne the bit */
  if(R(input) != R(output)) {
    /* Check the header and move i      ck if necessary */
    if(nxt_motor_get_count(READ_M     ) > 0 && R(R_state) == READ_IDLE) {
      W(R_state, READ_MOVE_HEADER     KWARD);
    }

    /* Check the header and fli     e bit if it is safe to     /
    if(nxt_motor_get_count(READ     OR) <= 0 && R(W_state) =     ITE_IDLE) {
      W(W_state, WRITE_FLIP);
    }
  } else {
    /* Nothing to change for w     er */
    W(W_state, WRITE_IDLE);
    C_state = C_MOVE;
  }
  break;
```

$\theta <= 0$

```
  case WRITE_FLIP:
#ifdef VERIFICATION
    /* Property 3: When writer flips a
       motor should be stopped. */

    /* FAILED!! with BOUND 120 */
    assert(R(T_speed) == 0 && R(R_speed) == 0);
#endif
```

NO!
The position of READ header is in safe area (<=0), however it's possible that it is **still moving**!

Properties

Property 2: When writer flips a bit, the tape motor and read motor should **stop**.

Controller Task

```
case C_WRITE:

  /* Check if we need to chagne the bit */
  if(R(input) != R(output)) {
    /* Check the header and move i   ck if necessary */
    if(nxt_motor_get_count(READ_M   ) > 0 && R(R_state) == READ_IDLE) {
      W(R_state, READ_MOVE_HEADER   KWARD);
    }

    /* Check the header and fli   e bit if it is safe to   /
    if(nxt_motor_get_count(READ   OR) <= 0 && R(W_state) =   ITE_IDLE) {
      W(W_state, WRITE_FLIP);
    }
  } else {
    /* Nothing to change for w   er */
    W(W_state, WRITE_IDLE);
    C_state = C_MOVE;
  }
  break;
```

writer

reader

$\theta <= 0$

```
  case WRITE_FLIP:
#ifdef VERIFICATION
    /* Property 3: When writer flips a b
       motor should be stopped. */

    /* FAILED!! with BOUND 120 */
    assert(R(T_speed) == 0 && R(R_speed) == 0);
#endif
```

REKH(out tool) can find a counterexample within 2mins.

Experimental Results

Table 1. Experimental results. OL and SL = # lines of code in the original C program and the generated sequentialization $\mathcal{S}$, respectively; GL = size of the GOTO program produced by CBMC; Var and Clause = # variables and clauses in the SAT instance, respectively; S = verification result – 'Y' for SAFE, 'N' for UNSAFE, and 'U' for timeout (12,000s); Time = verification time in sec.

Name	OL	monoSeq SL	monoSeq SAT Size GL	monoSeq SAT Size Var	monoSeq SAT Size Clause	monoSeq S	monoSeq Time (sec)	harmonicSeq SL	harmonicSeq SAT Size GL	harmonicSeq SAT Size Var	harmonicSeq SAT Size Clause	harmonicSeq S	harmonicSeq Time (sec)
1 hyper-period													
nxt.ok1	396	2,158	12K	128K	399K	Y	21.22	2,378	17K	110K	354K	Y	4.22
nxt.bug1	398	2,158	12K	128K	399K	N	6.22	2,378	17K	110K	354K	N	4.36
nxt.ok2	388	2,215	12K	132K	410K	Y	11.16	2,432	18K	111K	356K	Y	4.69
nxt.bug2	405	2,389	15K	135K	422K	N	8.66	2,704	23K	114K	372K	N	5.81
nxt.ok3	405	2,389	15K	135K	425K	Y	14.46	2,704	23K	109K	358K	Y	5.71
aso.bug1	421	2,557	17K	167K	541K	N	12.05	3,094	29K	173K	568K	N	6.67
aso.bug2	421	2,627	17K	167K	539K	N	11.61	3,184	29K	165K	549K	N	6.71
aso.ok1	418	2,561	17K	164K	525K	Y	22.20	3,098	28K	147K	486K	Y	6.51
aso.bug3	445	3,118	24K	350K	1,117K	N	22.15	4,131	41K	341K	1,108K	Y	19.27
aso.bug4	444	3,105	23K	325K	1,027K	N	16.32	4,118	40K	307K	1,001K	N	10.83
aso.ok2	443	3,106	23K	326K	1,035K	Y	601.59	4,119	40K	311K	1,006K	Y	21.94
4 hyper-periods													
nxt.ok1	396	14,014	57K	1,825K	5,816K	Y	1,305	2,393	71K	471K	1,610K	Y	70.59
nxt.bug1	398	14,014	57K	1,825K	5,816K	N	1,406	2,393	71K	471K	1,610K	N	73.27
nxt.ok2	388	14,156	60K	1,850K	5,849K	Y	1,382	2,447	73K	475K	1,618K	Y	67.08
nxt.bug2	405	14,573	71K	1,887K	5,978K	N	362	2,722	94K	485K	1,667K	N	77.39
nxt.ok3	405	14,573	71K	1,884K	5,964K	U	—	2,722	93K	466K	1,723K	Y	101.01
aso.bug1	421	14,942	81K	2,359K	7,699K	N	894	3,115	115K	726K	2,741K	N	143.52
aso.bug2	421	15,097	81K	2,359K	7,689K	N	773	3,205	116K	692K	2,438K	N	107.66
aso.ok1	418	14,946	80K	2,331K	7,590K	U	—	3,119	114K	620K	2,188K	Y	110.21
aso.bug3	445	16,024	113K	5,016K	16,162K	N	9,034	4,161	167K	1,406K	4,774K	Y	215.02
aso.bug4	444	16,055	108K	4,729K	15,141K	N	6,016	4,148	161K	1,271K	4,295K	N	168.22
aso.ok2	443	16,056	109K	4,734K	15,159K	U	—	4,149	162K	1,289K	4,360K	Y	200.25

Table 1. Experimental results. OL and SL = # lines of code in the original C program and the generated sequentialization $\mathcal{S}$, respectively; GL = size of the GOTO program produced by CBMC; Var and Clause = # variables and clauses in the SAT instance, respectively; S = verification result – 'Y' for SAFE, 'N' for UNSAFE, and 'U' for timeout (12,000s); Time = verification time in sec.

		monoSeq						harmonicSeq					
Name			SAT Size			S	Time		SAT Size			S	Time
	OL	SL	GL	Var	Clause		(sec)	SL	GL	Var	Clause		(sec)
1 hyper-period													
nxt.ok1	396	2,158	12K	128K	399K	Y	21.22	2,378	17K	110K	354K	Y	4.22
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		MONOSEQ						HARMONICSEQ					
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1 hyper-period													
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		monoSeq						harmonicSeq					
Name			SAT Size			S	Time		SAT Size			S	Time
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aso.bug1	421	14,942	81K	2,359K	7,699K	N	894	3,115	115K	726K	2,741K	N	143.52
aso.bug2	421	15,097	81K	2,359K	7,689K	N	773	3,205	116K	692K	2,438K	N	107.66
aso.ok1	418	14,946	80K	2,331K	7,590K	U	—	3,119	114K	620K	2,188K	Y	110.21
aso.bug3	445	16,024	113K	5,016K	16,162K	N	9,034	4,161	167K	1,406K	4,774K	Y	215.02
aso.bug4	444	16,055	108K	4,729K	15,141K	N	6,016	4,148	161K	1,271K	4,295K	N	168.22
aso.ok2	443	16,056	109K	4,734K	15,159K	U	—	4,149	162K	1,289K	4,360K	Y	200.25

Table 1. Experimental results. OL and SL = # lines of code in the original C program and the generated sequentialization $\mathcal{S}$, respectively; GL = size of the GOTO program produced by CBMC; Var and Clause = # variables and clauses in the SAT instance, respectively; S = verification result – ‘Y’ for SAFE, ‘N’ for UNSAFE, and ‘U’ for timeout (12,000s); Time = verification time in sec.

		monoSeq						harmonicSeq					
Name			SAT Size			S	Time		SAT Size			S	Time
	OL	SL	GL	Var	Clause		(sec)	SL	GL	Var	Clause		(sec)
1 hyper-period													
nxt.ok1	396	2,158	12K	128K	399K	Y	21.22	2,378	17K	110K	354K	Y	4.22
nxt.bug1	398	2,158	12K	128K	399K	N	6.22	2,378	17K	110K	354K	N	4.36
nxt.ok2	388	2,215	12K	132K	410K	Y	11.16	2,432	18K	111K	356K	Y	4.69
nxt.bug2	405	2,389	15K	135K	422K	N	8.66	2,704	23K	114K	372K	N	5.81
nxt.ok3	405	2,389	15K	135K	425K	Y	14.46	2,704	23K	109K	358K	Y	5.71
aso.bug1	421	2,557	17K	167K	541K	N	12.05	3,094	29K	173K	568K	N	6.67
aso.bug2	421	2,627	17K	167K	539K	N	11.61	3,184	29K	165K	549K	N	6.71
aso.ok1	418	2,561	17K	164K	525K	Y	22.20	3,098	28K	147K	486K	Y	6.51
aso.bug3	445	3,118	24K	350K	1,117K	N	22.15	4,131	41K	341K	1,108K	Y	19.27
aso.bug4	444	3,105	23K	325K	1,027K	N	16.32	4,118	40K	307K	1,001K	N	10.83
aso.ok2	443	3,106	23K	326K	1,035K	Y	601.59	4,119	40K	311K	1,006K	Y	21.94
4 hyper-periods													
nxt.ok1	396	14,014	57K	1,825K	5,816K	Y	1,305	2,393	71K	471K	1,610K	Y	70.59
nxt.bug1	398	14,014	57K	1,825K	5,816K	N	1,406	2,393	71K	471K	1,610K	N	73.27
nxt.ok2	388	14,156	60K	1,850K	5,849K	Y	1,382	2,447	73K	475K	1,618K	Y	67.08
nxt.bug2	405	14,573	71K	1,887K	5,978K	N	362	2,722	94K	485K	1,667K	N	77.39
nxt.ok3	405	14,573	71K	1,884K	5,964K	U	—	2,722	93K	466K	1,723K	Y	101.01
aso.bug1	421	14,942	81K	2,359K	7,699K	N	894	3,115	115K	726K	2,741K	N	143.52
aso.bug2	421	15,097	81K	2,359K	7,689K	N	773	3,205	116K	692K	2,438K	N	107.66
aso.ok1	418	14,946	80K	2,331K	7,590K	U	—	3,119	114K	620K	2,188K	Y	110.21
aso.bug3	445	16,024	113K	5,016K	16,162K	N	9,034	4,161	167K	1,406K	4,774K	Y	215.02
aso.bug4	444	16,055	108K	4,729K	15,141K	N	6,016	4,148	161K	1,271K	4,295K	N	168.22
aso.ok2	443	16,056	109K	4,734K	15,159K	U	—	4,149	162K	1,289K	4,360K	Y	200.25

Table 1. Experimental results. OL and SL = # lines of code in the original C program and the generated sequentialization $\mathcal{S}$, respectively; GL = size of the GOTO program produced by CBMC; Var and Clause = # variables and clauses in the SAT instance, respectively; S = verification result – 'Y' for SAFE, 'N' for UNSAFE, and 'U' for timeout (12,000s); Time = verification time in sec.

Name	OL	monoSeq SL	monoSeq SAT Size GL	monoSeq SAT Size Var	monoSeq SAT Size Clause	monoSeq S	monoSeq Time (sec)	harmonicSeq SL	harmonicSeq SAT Size GL	harmonicSeq SAT Size Var	harmonicSeq SAT Size Clause	harmonicSeq S	harmonicSeq Time (sec)
1 hyper-period													
nxt.ok1	396	2,158	12K	128K	399K	Y	21.22	2,378	17K	110K	354K	Y	4.22
nxt.bug1	398	2,158	12K	128K	399K	N	6.22	2,378	17K	110K	354K	N	4.36
nxt.ok2	388	2,215	12K	132K	410K	Y	11.16	2,432	18K	111K	356K	Y	4.69
nxt.bug2	405	2,389	15K	135K	422K	N	8.66	2,704	23K	114K	372K	N	5.81
nxt.ok3	405	2,389	15K	135K	425K	Y	14.46	2,704	23K	109K	358K	Y	5.71
aso.bug1	421	2,557	17K	167K	541K	N	12.05	3,094	29K	173K	568K	N	6.67
aso.bug2	421	2,627	17K	167K	539K	N	11.61	3,184	29K	165K	549K	N	6.71
aso.ok1	418	2,561	17K	164K	525K	Y	22.20	3,098	28K	147K	486K	Y	6.51
aso.bug3	445	3,118	24K	350K	1,117K	N	22.15	4,131	41K	341K	1,108K	Y	19.27
aso.bug4	444	3,105	23K	325K	1,027K	N	16.32	4,118	40K	307K	1,001K	N	10.83
aso.ok2	443	3,106	23K	326K	1,035K	Y	601.59	4,119	40K	311K	1,006K	Y	21.94
4 hyper-periods													
nxt.ok1	396	14,014	57K	1,825K	5,816K	Y	1,305	2,393	71K	471K	1,610K	Y	70.59
nxt.bug1	398	14,014	57K	1,825K	5,816K	N	1,406	2,393	71K	471K	1,610K	N	73.27
nxt.ok2	388	14,156	60K	1,850K	5,849K	Y	1,382	2,447	73K	475K	1,618K	Y	67.08
nxt.bug2	405	14,573	71K	1,887K	5,978K	N	362	2,722	94K	485K	1,667K	N	77.39
nxt.ok3	405	14,573	71K	1,884K	5,964K	U	—	2,722	93K	466K	1,723K	Y	101.01
aso.bug1	421	14,942	81K	2,359K	7,699K	N	894	3,115	115K	726K	2,741K	N	143.52
aso.bug2	421	15,097	81K	2,359K	7,689K	N	773	3,205	116K	692K	2,438K	N	107.66
aso.ok1	418	14,946	80K	2,331K	7,590K	U	—	3,119	114K	620K	2,188K	Y	110.21
aso.bug3	445	16,024	113K	5,016K	16,162K	N	9,034	4,161	167K	1,406K	4,774K	Y	215.02
aso.bug4	444	16,055	108K	4,729K	15,141K	N	6,016	4,148	161K	1,271K	4,295K	N	168.22
aso.ok2	443	16,056	109K	4,734K	15,159K	U	—	4,149	162K	1,289K	4,360K	Y	200.25

Table 1. Experimental results. OL and SL = # lines of code in the original C program and the generated sequentialization $\mathcal{S}$, respectively; GL = size of the GOTO program produced by CBMC; Var and Clause = # variables and clauses in the SAT instance, respectively; S = verification result – 'Y' for SAFE, 'N' for UNSAFE, and 'U' for timeout (12,000s); Time = verification time in sec.

		MONOSEQ						HARMONICSEQ					
Name			SAT Size			S	Time		SAT Size			S	Time
	OL	SL	GL	Var	Clause		(sec)	SL	GL	Var	Clause		(sec)
1 hyper-period													
nxt.ok1	396	2,158	12K	128K	399K	Y	21.22	2,378	17K	110K	354K	Y	4.22
nxt.bug1	398	2,158	12K	128K	399K	N	6.22	2,378	17K	110K	354K	N	4.36
nxt.ok2	388	2,215	12K	132K	410K	Y	11.16	2,432	18K	111K	356K	Y	4.69
nxt.bug2	405	2,389	15K	135K	422K	N	8.66	2,704	23K	114K	372K	N	5.81
nxt.ok3	405	2,389	15K	135K	425K	Y	14.46	2,704	23K	109K	358K	Y	5.71
aso.bug1	421	2,557	17K	167K	541K	N	12.05	3,094	29K	173K	568K	N	6.67
aso.bug2	421	2,627	17K	167K	539K	N	11.61	3,184	29K	165K	549K	N	6.71
aso.ok1	418	2,561	17K	164K	525K	Y	22.20	3,098	28K	147K	486K	Y	6.51
aso.bug3	445	3,118	24K	350K	1,117K	N	22.15	4,131	41K	341K	1,108K	Y	19.27
aso.bug4	444	3,105	23K	325K	1,027K	N	16.32	4,118	40K	307K	1,001K	N	10.83
aso.ok2	443	3,106	23K	326K	1,035K	Y	601.59	4,119	40K	311K	1,006K	Y	21.94
4 hyper-periods													
nxt.ok1	396	14,014	57K	1,825K	5,816K	Y	1,305	2,393	71K	471K	1,610K	Y	70.59
nxt.bug1	398	14,014	57K	1,825K	5,816K	N	1,406	2,393	71K	471K	1,610K	N	73.27
nxt.ok2	388	14,156	60K	1,850K	5,849K	Y	1,382	2,447	73K	475K	1,618K	Y	67.08
nxt.bug2	405	14,573	71K	1,887K	5,978K	N	362	2,722	94K	485K	1,667K	N	77.39
nxt.ok3	405	14,573	71K	1,884K	5,964K	U	—	2,722	93K	466K	1,723K	Y	101.01
aso.bug1	421	14,942	81K	2,359K	7,699K	N	894	3,115	115K	726K	2,741K	N	143.52
aso.bug2	421	15,097	81K	2,359K	7,689K	N	773	3,205	116K	692K	2,438K	N	107.66
aso.ok1	418	14,946	80K	2,331K	7,590K	U	—	3,119	114K	620K	2,188K	Y	110.21
aso.bug3	445	16,024	113K	5,016K	16,162K	N	9,034	4,161	167K	1,406K	4,774K	Y	215.02
aso.bug4	444	16,055	108K	4,729K	15,141K	N	6,016	4,148	161K	1,271K	4,295K	N	168.22
aso.ok2	443	16,056	109K	4,734K	15,159K	U	—	4,149	162K	1,289K	4,360K	Y	200.25

Table 1. Experimental results. OL and SL = # lines of code in the original C program and the generated sequentialization $\mathcal{S}$, respectively; GL = size of the GOTO program produced by CBMC; Var and Clause = # variables and clauses in the SAT instance, respectively; S = verification result – 'Y' for SAFE, 'N' for UNSAFE, and 'U' for timeout (12,000s); Time = verification time in sec.

		monoSeq						harmonicSeq					
Name	OL	SL	SAT Size GL	SAT Size Var	SAT Size Clause	S	Time (sec)	SL	SAT Size GL	SAT Size Var	SAT Size Clause	S	Time (sec)
1 hyper-period													
nxt.ok1	396	2,158	12K	128K	399K	Y	21.22	2,378	17K	110K	354K	Y	4.22
nxt.bug1	398	2,158	12K	128K	399K	N	6.22	2,378	17K	110K	354K	N	4.36
nxt.ok2	388	2,215	12K	132K	410K	Y	11.16	2,432	18K	111K	356K	Y	4.69
nxt.bug2	405	2,389	15K	135K	422K	N	8.66	2,704	23K	114K	372K	N	5.81
nxt.ok3	405	2,389	15K	135K	425K	Y	14.46	2,704	23K	109K	358K	Y	5.71
aso.bug1	421	2,557	17K	167K	541K	N	12.05	3,094	29K	173K	568K	N	6.67
aso.bug2	421	2,627	17K	167K	539K	N	11.61	3,184	29K	165K	549K	N	6.71
aso.ok1	418	2,561	17K	164K	525K	Y	22.20	3,098	28K	147K	486K	Y	6.51
aso.bug3	445	3,118	24K	350K	1,117K	N	22.15	4,131	41K	341K	1,108K	Y	19.27
aso.bug4	444	3,105	23K	325K	1,027K	N	16.32	4,118	40K	307K	1,001K	N	10.83
aso.ok2	443	3,106	23K	326K	1,035K	Y	601.59	4,119	40K	311K	1,006K	Y	21.94
4 hyper-periods													
nxt.ok1	396	14,014	57K	1,825K	5,816K	Y	1,305	2,393	71K	471K	1,610K	Y	70.59
nxt.bug1	398	14,014	57K	1,825K	5,816K	N	1,406	2,393	71K	471K	1,610K	N	73.27
nxt.ok2	388	14,156	60K	1,850K	5,849K	Y	1,382	2,447	73K	475K	1,618K	Y	67.08
nxt.bug2	405	14,573	71K	1,887K	5,978K	N	362	2,722	94K	485K	1,667K	N	77.39
nxt.ok3	405	14,573	71K	1,884K	5,964K	U	—	2,722	93K	466K	1,723K	Y	101.01
aso.bug1	421	14,942	81K	2,359K	7,699K	N	894	3,115	115K	726K	2,741K	N	143.52
aso.bug2	421	15,097	81K	2,359K	7,689K	N	773	3,205	116K	692K	2,438K	N	107.66
aso.ok1	418	14,946	80K	2,331K	7,590K	U	—	3,119	114K	620K	2,188K	Y	110.21
aso.bug3	445	16,024	113K	5,016K	16,162K	N	9,034	4,161	167K	1,406K	4,774K	Y	215.02
aso.bug4	444	16,055	108K	4,729K	15,141K	N	6,016	4,148	161K	1,271K	4,295K	N	168.22
aso.ok2	443	16,056	109K	4,734K	15,159K	U	—	4,149	162K	1,289K	4,360K	Y	200.25

Table 1. Experimental results. OL and SL = # lines of code in the original C program and the generated sequentialization $\mathcal{S}$, respectively; GL = size of the GOTO program produced by CBMC; Var and Clause = # variables and clauses in the SAT instance, respectively; S = verification result – 'Y' for SAFE, 'N' for UNSAFE, and 'U' for timeout (12,000s); Time = verification time in sec.

		monoSeq						harmonicSeq					
Name			SAT Size			S	Time		SAT Size			S	Time
	OL	SL	GL	Var	Clause		(sec)	SL	GL	Var	Clause		(sec)
1 hyper-period													
nxt.ok1	396	2,158	12K	128K	399K	Y	21.22	2,378	17K	110K	354K	Y	4.22
nxt.bug1	398	2,158	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	354K	N	4.36
nxt.ok2	388	2,215	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	356K	Y	4.69
nxt.bug2	405	2,389	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	372K	N	5.81
nxt.ok3	405	2,389	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	358K	Y	5.71
aso.bug1	421	2,557	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	568K	N	6.67
aso.bug2	421	2,627	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	549K	N	6.71
aso.ok1	418	2,561	17K	164K	525K	Y	22.20	3,098	28K	147K	486K	Y	6.51
aso.bug3	445	3,118	24K	350K	1,117K	N	22.15	4,131	41K	341K	1,108K	Y	19.27
aso.bug4	444	3,105	23K	325K	1,027K	N	16.32	4,118	40K	307K	1,001K	N	10.83
aso.ok2	443	3,106	23K	326K	1,035K	Y	601.59	4,119	40K	311K	1,006K	Y	21.94
4 hyper-periods													
nxt.ok1	396	14,014	57K	1,825K	5,816K	Y	1,305	2,393	71K	471K	1,610K	Y	70.59
nxt.bug1	398	14,014	57K	1,825K	5,816K	N	1,406	2,393	71K	471K	1,610K	N	73.27
nxt.ok2	388	14,156	60K	1,850K	5,849K	Y	1,382	2,447	73K	475K	1,618K	Y	67.08
nxt.bug2	405	14,573	71K	1,887K	5,978K	N	362	2,722	94K	485K	1,667K	N	77.39
nxt.ok3	405	14,573	71K	1,884K	5,964K	U	—	2,722	93K	466K	1,723K	Y	101.01
aso.bug1	421	14,942	81K	2,359K	7,699K	N	894	3,115	115K	726K	2,741K	N	143.52
aso.bug2	421	15,097	81K	2,359K	7,689K	N	773	3,205	116K	692K	2,438K	N	107.66
aso.ok1	418	14,946	80K	2,331K	7,590K	U	—	3,119	114K	620K	2,188K	Y	110.21
aso.bug3	445	16,024	113K	5,016K	16,162K	N	9,034	4,161	167K	1,406K	4,774K	Y	215.02
aso.bug4	444	16,055	108K	4,729K	15,141K	N	6,016	4,148	161K	1,271K	4,295K	N	168.22
aso.ok2	443	16,056	109K	4,734K	15,159K	U	—	4,149	162K	1,289K	4,360K	Y	200.25

MonoSeq considers **infeasible** thread executions and declares the program unsafe (False Alarm)

Table 2. Experimental results of concurrent Turing Machine. H = # of hyper-periods, OL and SL = # lines of code in the original C program and the generated sequentialization $\mathcal{S}$, respectively; GL = size of the GOTO program produced by CBMC; Var and Clause = # variables and clauses in the SAT instance, respectively; S = verification result – 'Y' for SAFE, 'N' for UNSAFE, and 'U' for timeout (85,000s); Time = verification time in sec.

Name	H	OL	monoSeq SL	monoSeq SAT Size GL	monoSeq SAT Size Var	monoSeq SAT Size Clause	monoSeq S	monoSeq Time (sec)	harmonicSeq SL	harmonicSeq SAT Size GL	harmonicSeq SAT Size Var	harmonicSeq SAT Size Clause	harmonicSeq S	harmonicSeq Time (sec)
ctm.ok1	4	613	13K	121K	2,737K	8,774K	Y	44,781	7K	111K	1,063K	3,497K	Y	93.39
ctm.ok2	4	610	13K	119K	2,728K	8,738K	Y	21,804	7K	109K	1,055K	3,467K	Y	87.60
ctm.bug2	4	611	13K	118K	2,707K	8,674K	N	2,281	7K	108K	1,047K	3,441K	N	86.18
ctm.ok3	6	612	20K	222K	6,276K	20,163K	U	—	7K	171K	1,667K	5,566K	Y	243.76
ctm.bug3	6	612	20K	214K	5,914K	19,044K	N	84,625	7K	165K	1,609K	5,383K	N	248.65
ctm.ok4	8	613	29K	333K	10,390K	33,550K	U	—	7K	222K	2,178K	7,417K	Y	534.38

Table 2. Experimental results of concurrent Turing Machine. H = # of hyper-periods, OL and SL = # lines of code in the original C program and the generated sequentialization $\mathcal{S}$, respectively; GL = size of the GOTO program produced by CBMC; Var and Clause = # variables and clauses in the SAT instance, respectively; S = verification result – 'Y' for SAFE, 'N' for UNSAFE, and 'U' for timeout (85,000s); Time = verification time in sec.

			monoSeq						harmonicSeq					
Name	H	OL	SL	SAT Size GL	Var	Clause	S	Time (sec)	SL	SAT Size GL	Var	Clause	S	Time (sec)
ctm.ok1	4	613	13K	121K	2,737K	8,774K	Y	44,781	7K	111K	1,063K	3,497K	Y	93.39
ctm.ok2	4	610	13K	119K	2,728K	8,738K	Y	21,804	7K	109K	1,055K	3,467K	Y	87.60
ctm.bug2	4	611	13K	118K	2,707K	8,674K	N	2,281	7K	108K	1,047K	3,441K	N	86.18
ctm.ok3	6	612	20K	222K	6,276K	20,163K	U	—	7K	171K	1,667K	5,566K	Y	243.76
ctm.bug3	6	612	20K	214K	5,914K	19,044K	N	84,625	7K	165K	1,609K	5,383K	N	248.65
ctm.ok4	8	613	29K	333K	10,390K	33,550K	U	—	7K	222K	2,178K	7,417K	Y	534.38

Table 2. Experimental results of concurrent Turing Machine. H = # of hyper-periods, OL and SL = # lines of code in the original C program and the generated sequentialization $\mathcal{S}$, respectively; GL = size of the GOTO program produced by CBMC; Var and Clause = # variables and clauses in the SAT instance, respectively; S = verification result – 'Y' for SAFE, 'N' for UNSAFE, and 'U' for timeout (85,000s); Time = verification time in sec.

			monoSeq						harmonicSeq					
Name				SAT Size			S	Time		SAT Size			S	Time
	H	OL	SL	GL	Var	Clause		(sec)	SL	GL	Var	Clause		(sec)
ctm.ok1	4	613	13K	121K	2,737K	8,774K	Y	44,781	7K	111K	1,063K	3,497K	Y	93.39
ctm.ok2	4	610	13K	119K	2,728K	8,738K	Y	21,804	7K	109K	1,055K	3,467K	Y	87.60
ctm.bug2	4	611	13K	118K	2,707K	8,674K	N	2,281	7K	108K	1,047K	3,441K	N	86.18
ctm.ok3	6	612	20K	222K	6,276K	20,163K	U	—	7K	171K	1,667K	5,566K	Y	243.76
ctm.bug3	6	612	20K	214K	5,914K	19,044K	N	84,625	7K	165K	1,609K	5,383K	N	248.65
ctm.ok4	8	613	29K	333K	10,390K	33,550K	U	—	7K	222K	2,178K	7,417K	Y	534.38

480x Faster!

Thank you